Why is it so difficult to accelerate cosmic rays up to extreme energies? The Hillas criterium

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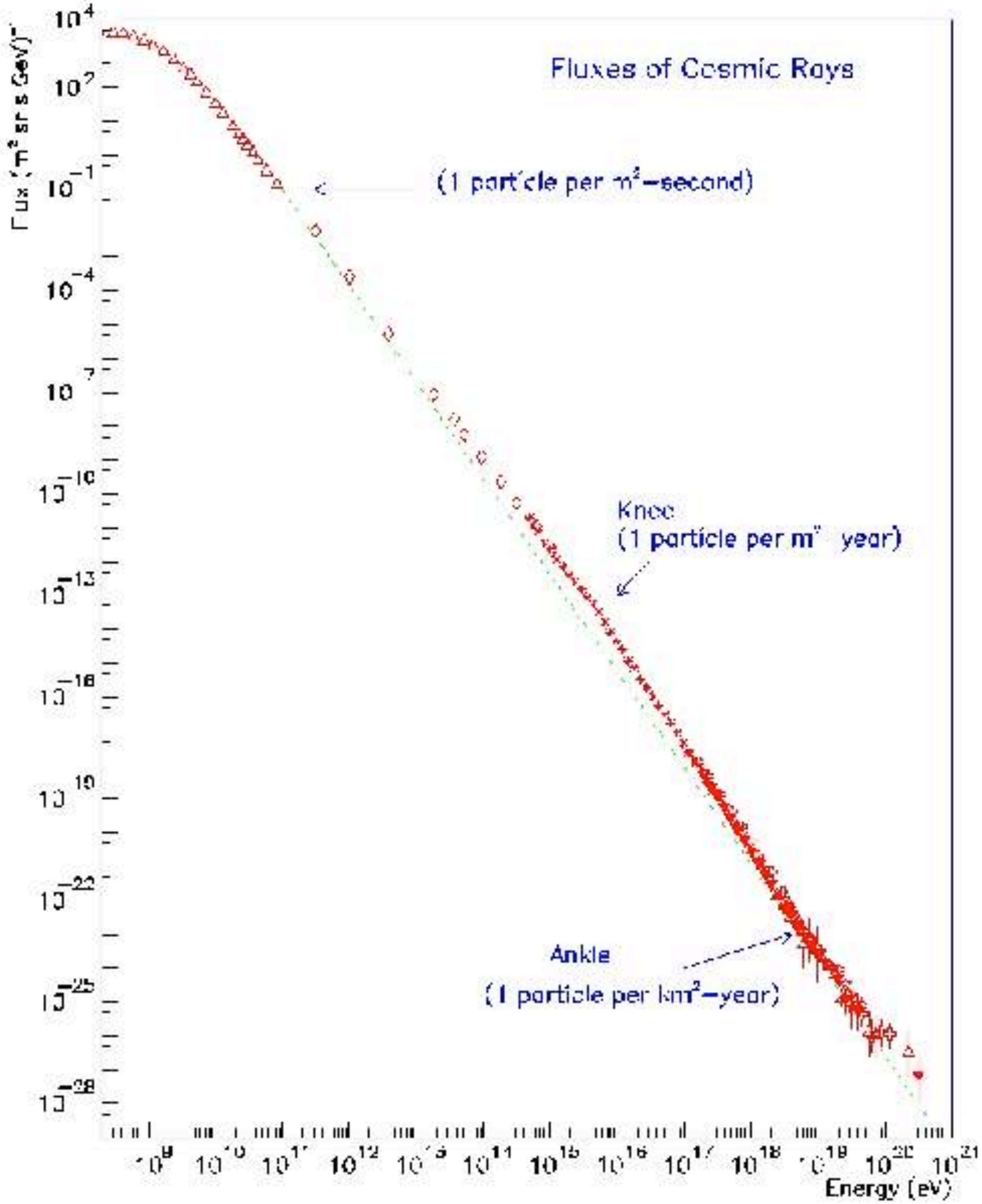
What are Cosmic Rays?

charged particles

Cosmic rays particles hit the Earth's atmosphere at the rate of about **1000 per square meter per second**. They are ionized nuclei - about **90% protons**, 9% alpha particles and the rest heavy nuclei - and they are distinguished by their high energies. Most cosmic rays are **relativistic**, having energies comparable or somewhat greater than their masses. A very few of them have ultrarelativistic energies extending up to 10^{20} eV (about 20 Joules), eleven order of magnitudes greater than the equivalent rest mass energy of a proton. The fundamental question of cosmic ray physics is, **"Where do they come from?"** and in particular, **"How are they accelerated to such high energies?"**.

Gaisser, Engel, Resconi "Cosmic Rays and Particle Physics"

The (local) Cosmic Ray spectrum



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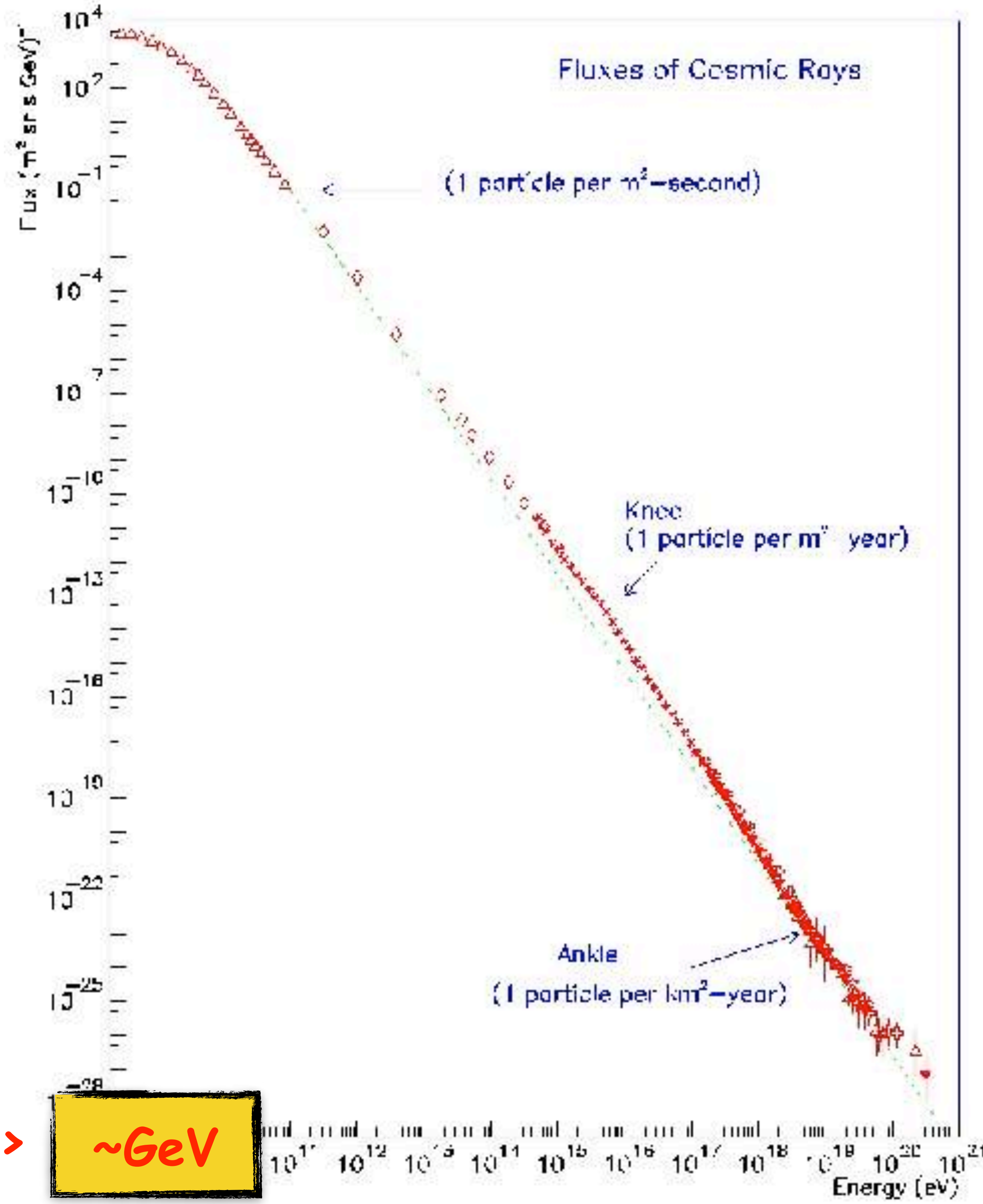

mean energy ->

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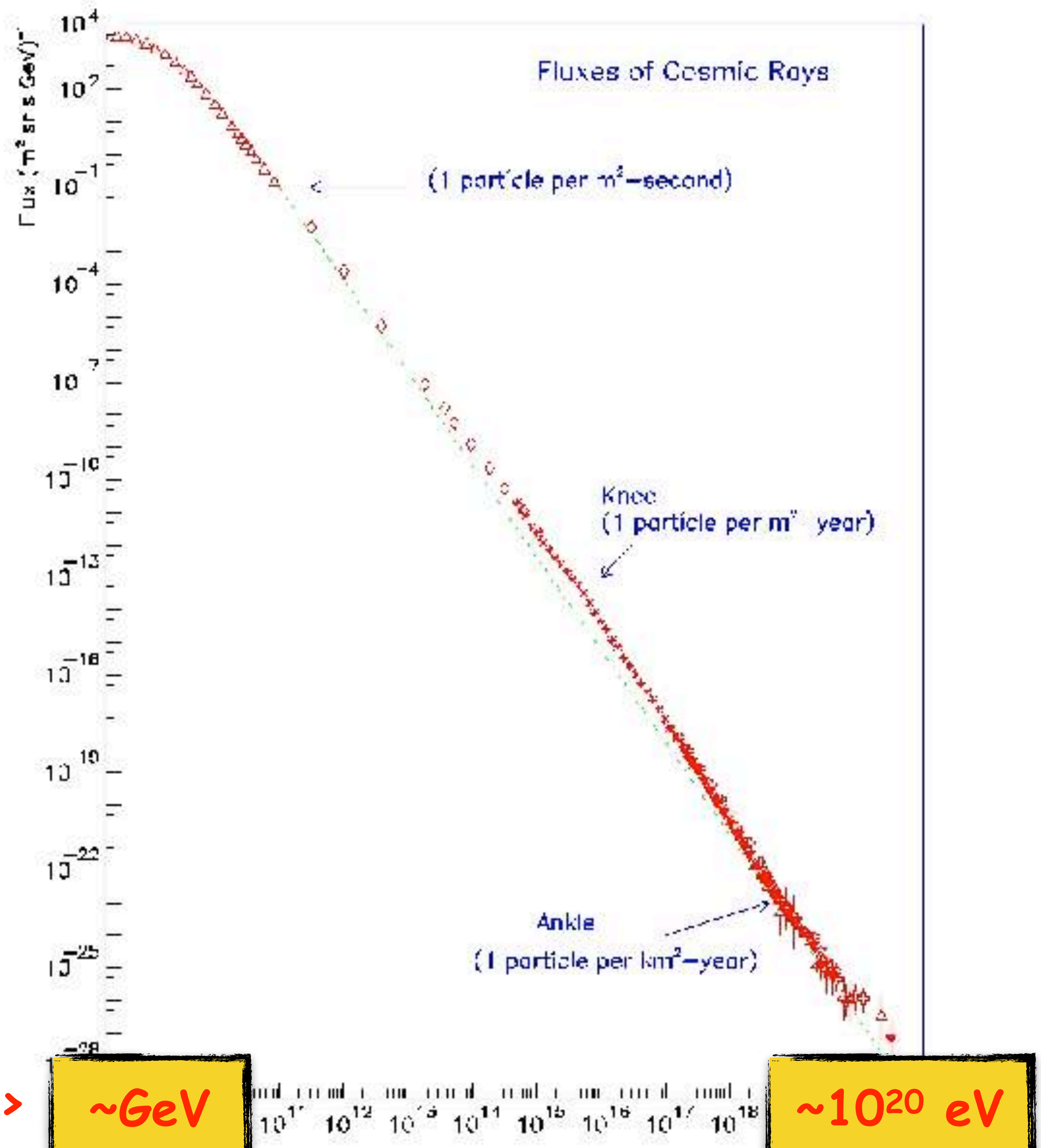
mean energy ->



<- maximum energy

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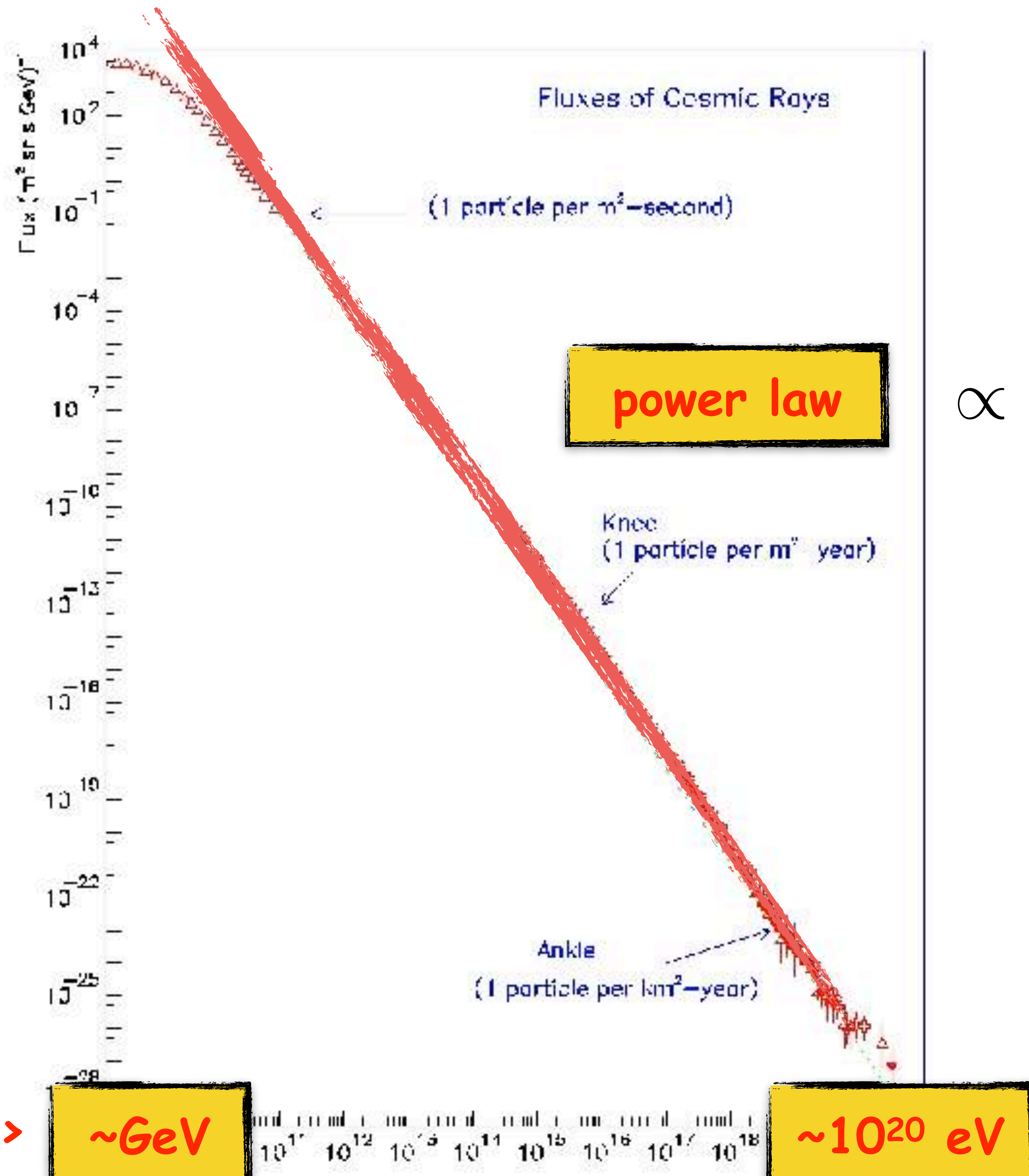
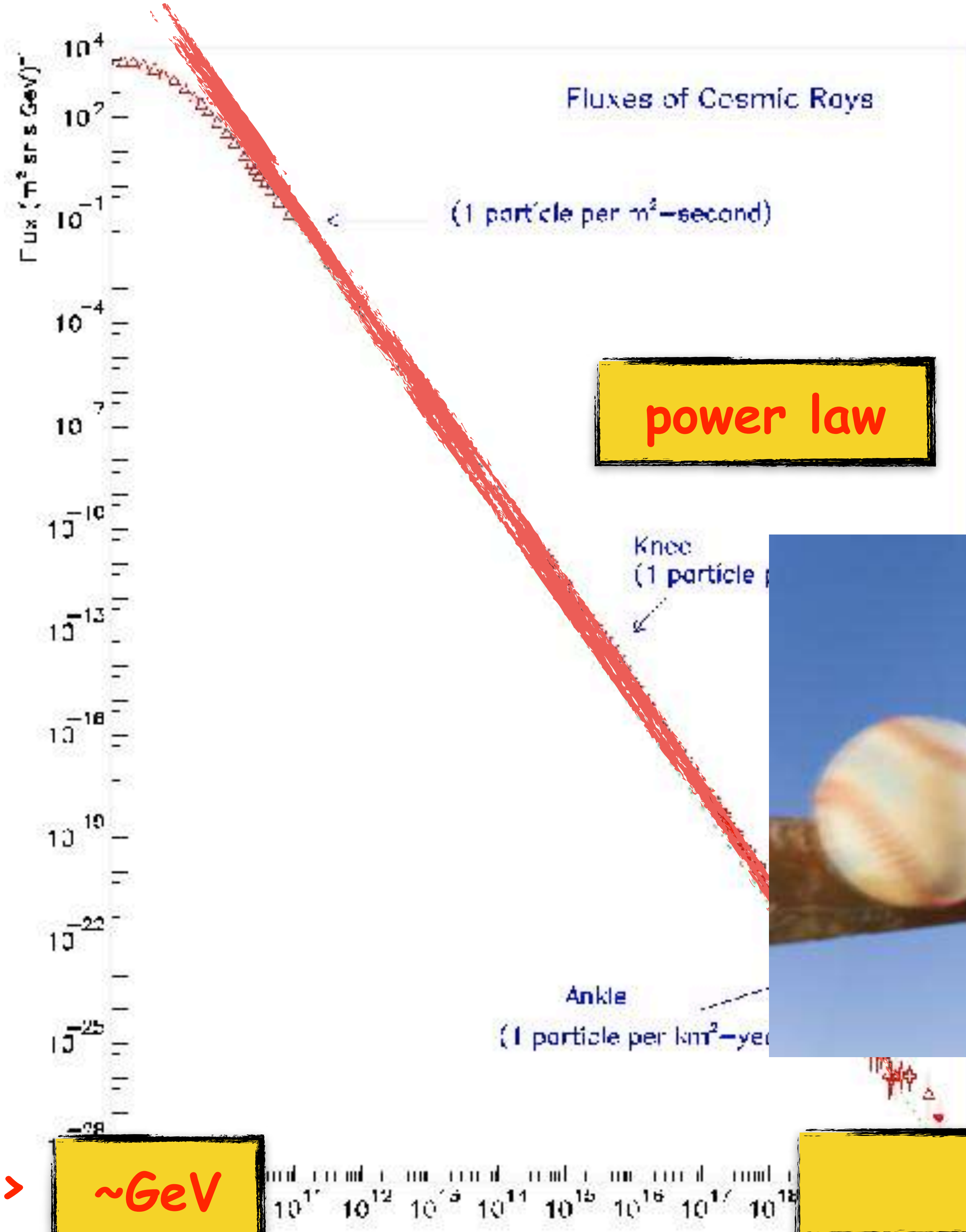


$$\propto E^{-2.7} - E^{-3}$$

mean energy ->

<- maximum energy

The (local) Cosmic Ray spectrum



power law

$$\propto E^{-2.7} - E^{-3}$$

mean energy -> ~GeV

tens of Joules!

Basics

The simplest accelerator

(The electrostatic accelerator)

Charged particles and electromagnetic fields

cosmic rays are charged particles —> they are affected by electromagnetic fields

$$\vec{E}(\vec{r},t) \qquad\qquad \vec{B}(\vec{r},t)$$

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A particle of charge q moving at a velocity u fill experience a force:

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = q\left(\vec{E} + \frac{\vec{u}}{c} \times \vec{B}\right)$$

relativistic momentum $\vec{p} = \gamma m \vec{u}$

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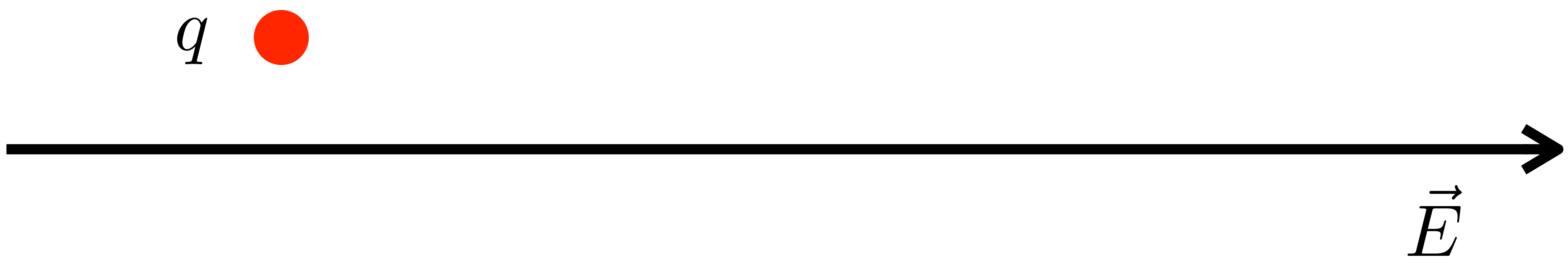
Lorentz force
$\perp$ to velocity —>
doesn't change
the particle energy!

relativistic momentum $\vec{p} = \gamma m \vec{u}$

The electrostatic accelerator

Problem: particle at rest at t = 0 and x = 0 embedded in a constant electric field



$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = q\vec{E}$$

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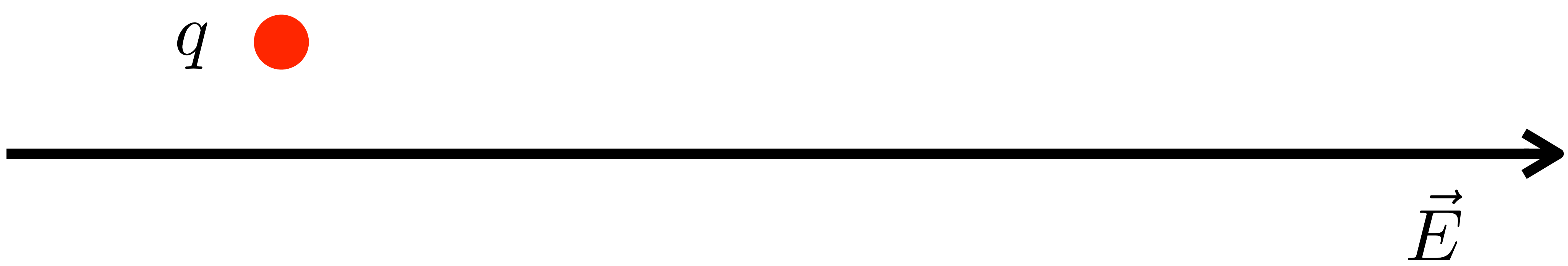

$$x = \frac{mc^2}{qE}\left\{\left[1+\left(\frac{qEt}{mc}\right)^2\right]^{1/2}-1\right\} \longrightarrow ct$$

$$\beta = \frac{\frac{qEt}{mc}}{\left[1+\left(\frac{qEt}{mc}\right)^2\right]^{1/2}} \longrightarrow 1$$

$$E_t = \left[1+\left(\frac{qEt}{mc}\right)^2\right]^{1/2} mc^2 \longrightarrow qEtc$$

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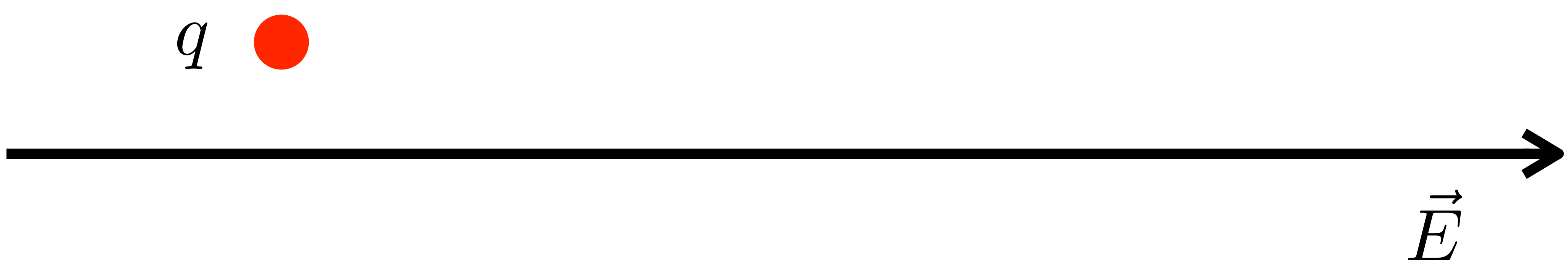

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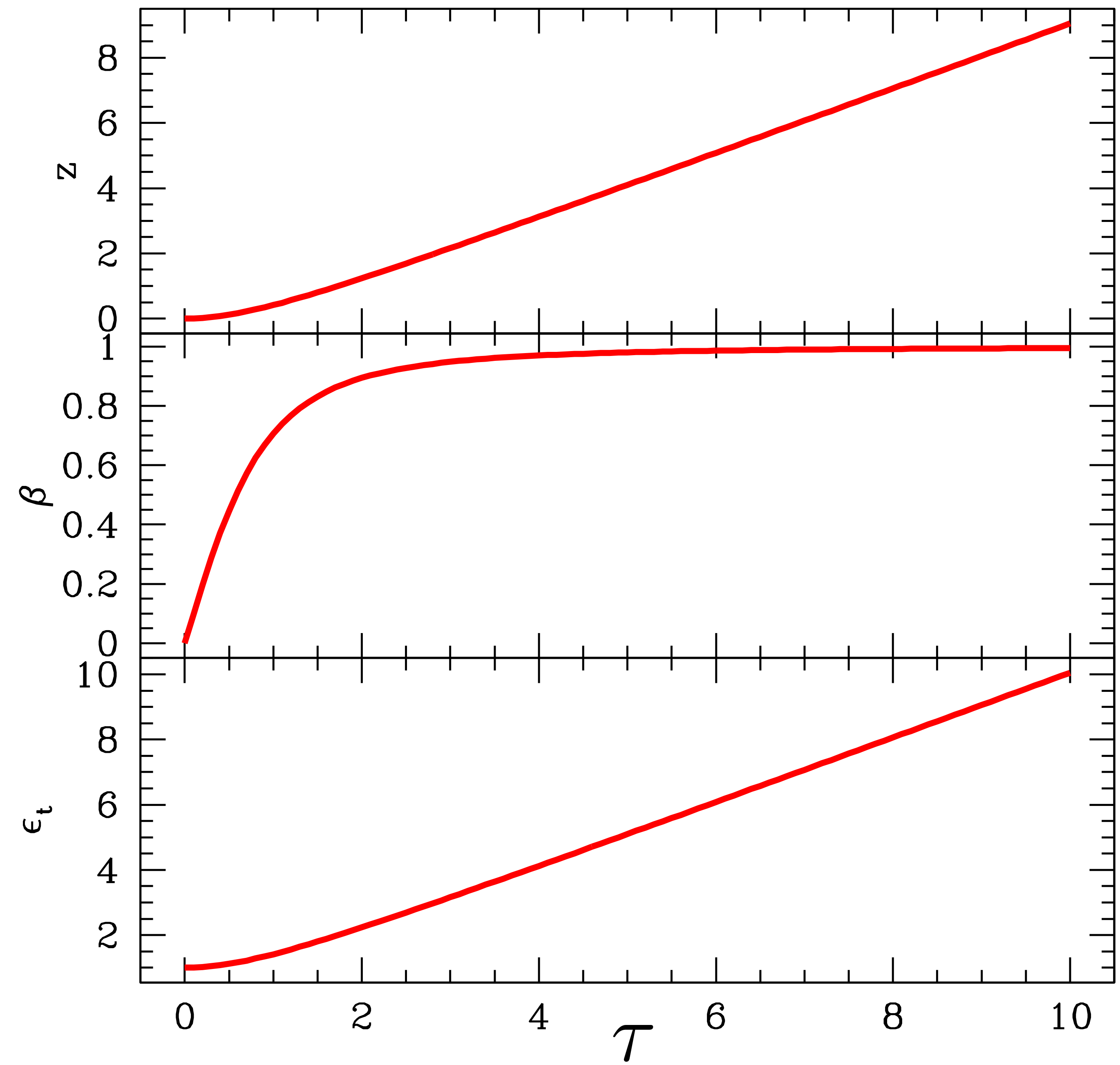
I use a subscript t not to confuse
particle energy with electric field

Non dimensional units

change variables

$$z = \frac{x}{\frac{mc^2}{qE}} \qquad \tau = \frac{t}{\frac{mc}{qE}} \qquad \epsilon_t = \frac{E_t}{mc^2}$$

$$\begin{aligned} z &= \left(1+\tau^2\right)^{1/2} - 1 \\ \beta &= \frac{\tau}{\left(1+\tau^2\right)^{1/2}} \\ \epsilon_t &= \left(1+\tau^2\right)^{1/2} \end{aligned}$$



no need to specify E

A useful expression

$$\begin{aligned} z &= \left(1+\tau^2\right)^{1/2} - 1 \\ \epsilon_t &= \left(1+\tau^2\right)^{1/2} \end{aligned}$$

$$\Rightarrow$$

$$\epsilon_t = z + 1$$

A useful expression

$$
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\epsilon_t &= \left(1+\tau^2\right)^{1/2}
\end{aligned}
$$

$$\epsilon_t = z + 1$$

in dimensional units:

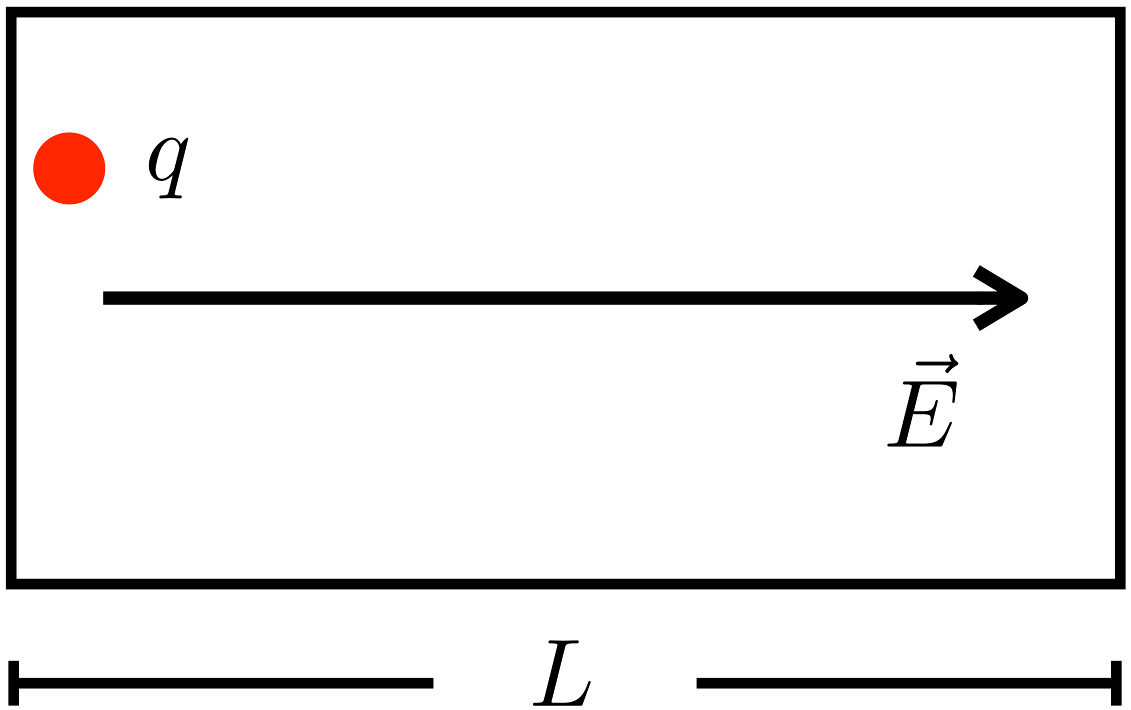
$$E_t = \left(\frac{qE}{mc^2}x + 1\right)mc^2$$

$$\frac{\mathrm{d}E_t}{\mathrm{d}x} = qE$$

constant rate

Maximum energy

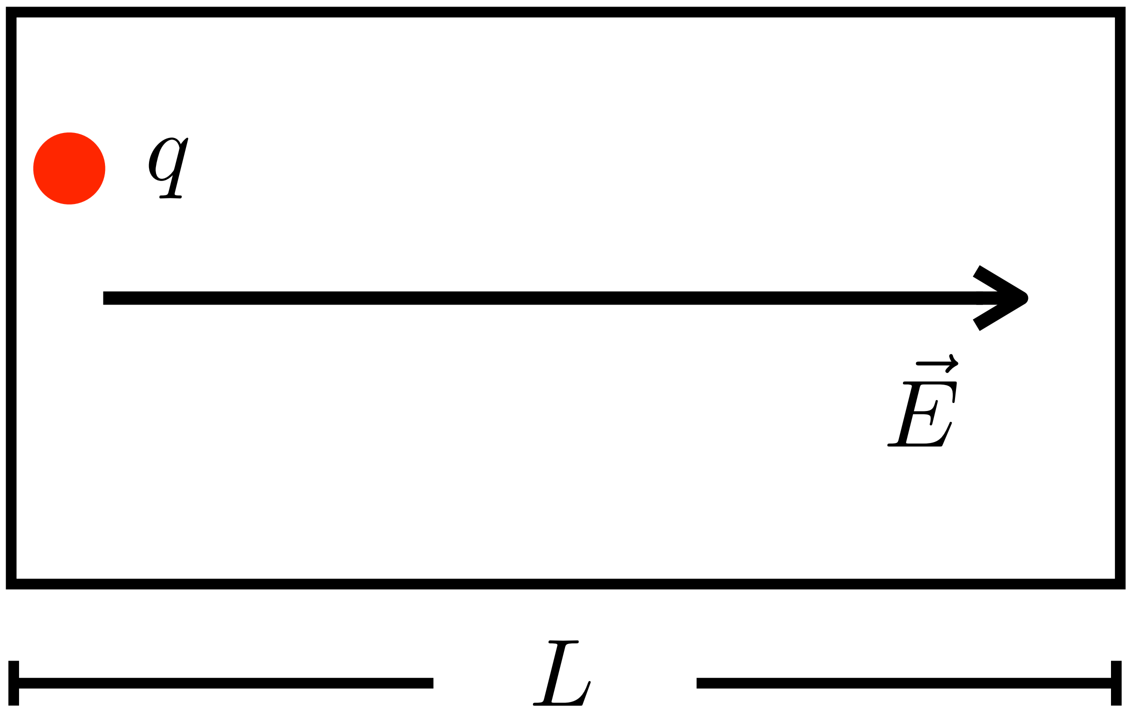
this is an accelerator



$$L \gg \frac{mc^2}{qE}$$

Maximum energy

this is an accelerator



$$L \gg \frac{mc^2}{qE}$$

$$E_t^{max} = qEtc = qEL$$

large accelerator

large charge

strong E field

The maximum energy problem: the role of radiative losses

Problem: accelerated charges emit radiation!

classic Larmor forula $\longrightarrow$ $$P = \frac{2}{3}\frac{q^2}{c^3}a^2$$ acceleration

Section 14.2 of Jackson

The maximum energy problem: the role of radiative losses

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relativistic generalisation

$$P = \frac{2}{3}\frac{q^2}{m^2c^3}\left[\left(\frac{\mathrm{d}\vec{p}}{\mathrm{d}\tau}\right)^2 - \beta^2\left(\frac{\mathrm{d}p}{\mathrm{d}\tau}\right)^2\right]$$

τ proper time —> $\mathrm{d}\tau = \mathrm{d}t/\gamma$

linear acceleration —> $\frac{\mathrm{d}\vec{p}}{\mathrm{d}\tau} = \frac{\mathrm{d}p}{\mathrm{d}\tau}$

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Section 14.2 of Jackson

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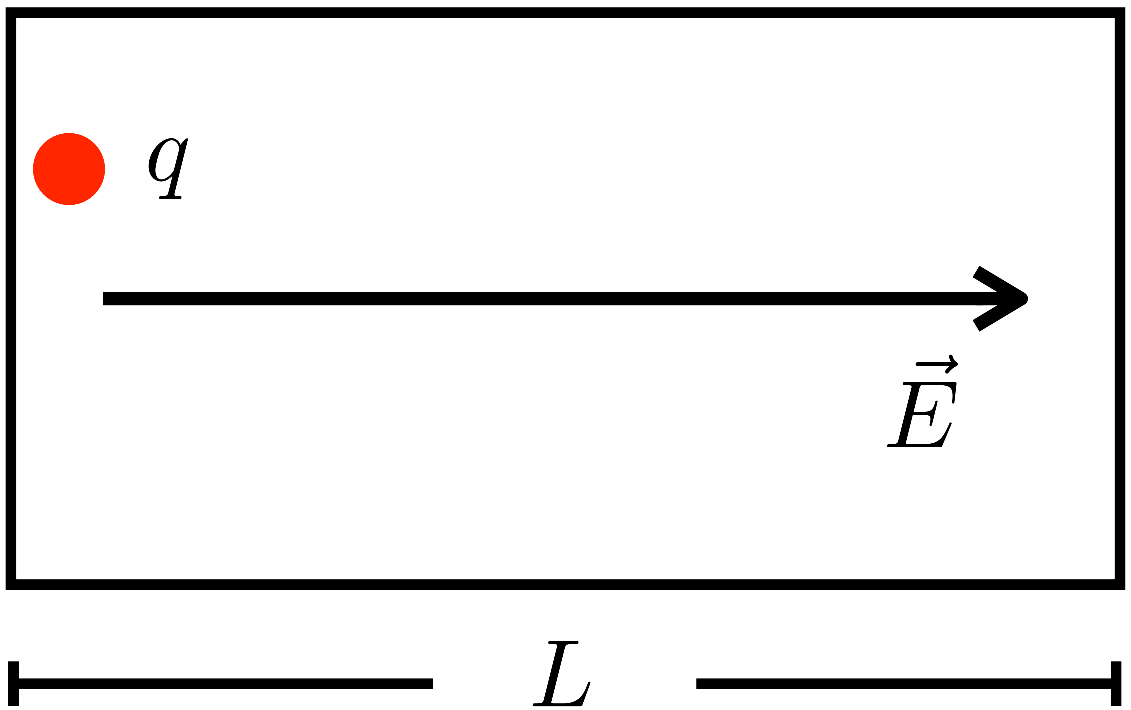
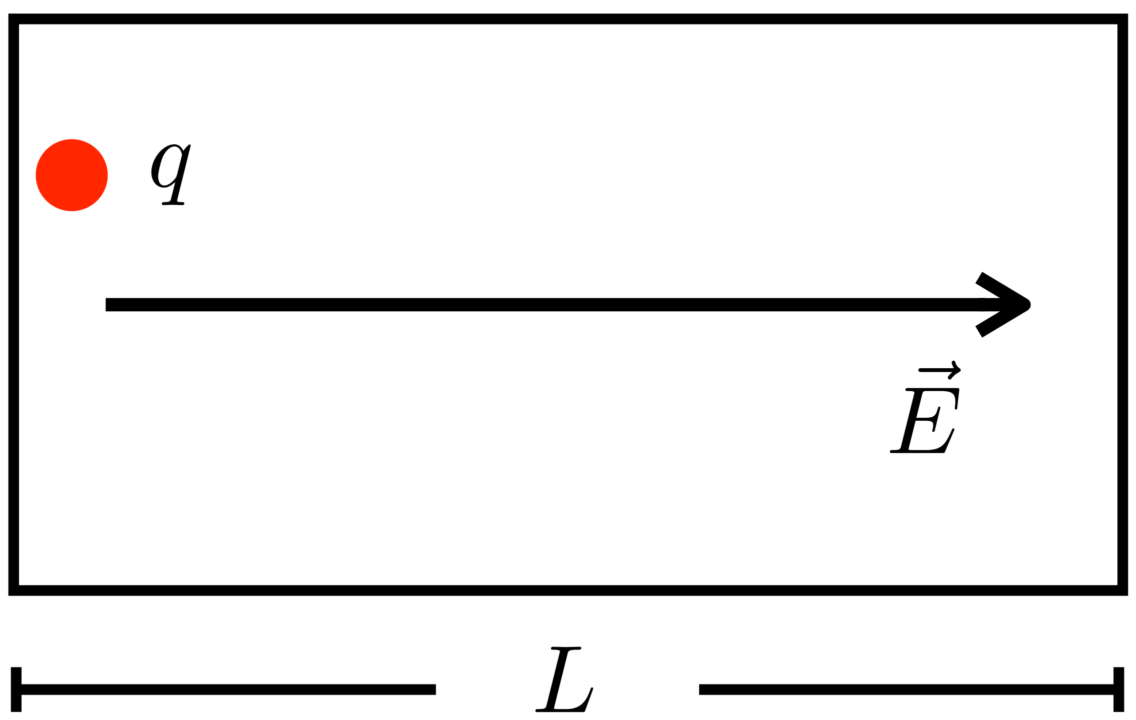


the particle gains energy at a rate $$\frac{\mathrm{d}E_t}{\mathrm{d}t}$$

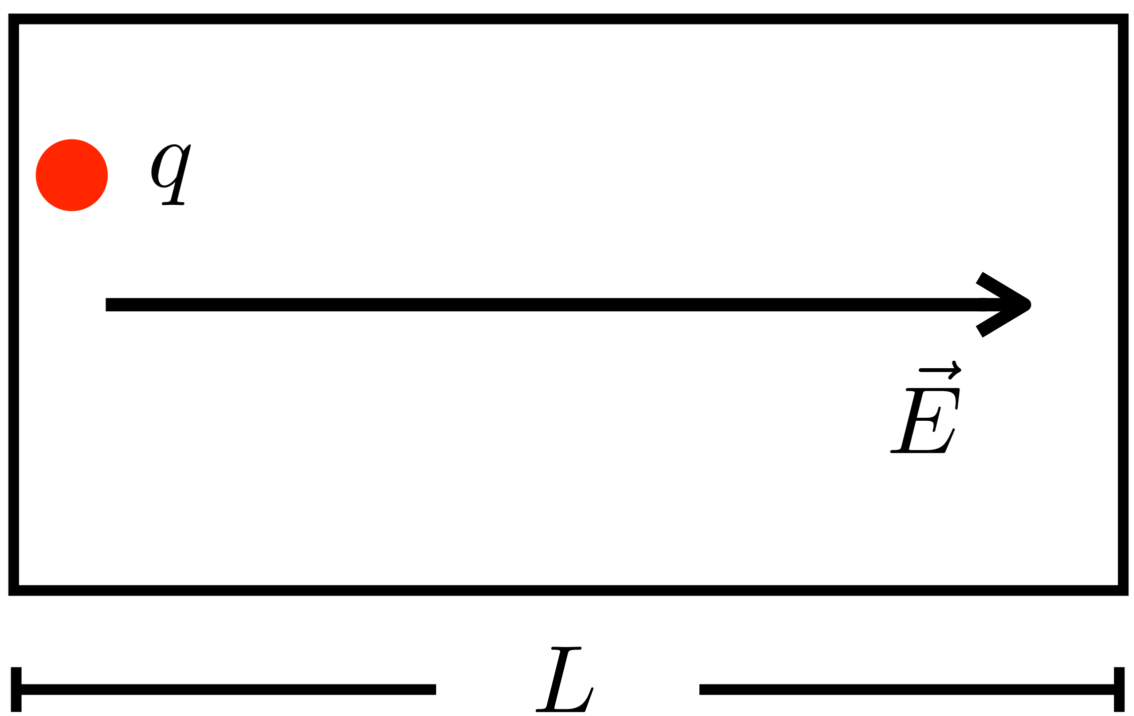
The maximum energy problem: the role of radiative losses



the particle gains energy at a rate $\dfrac{\mathrm{d}E_t}{\mathrm{d}t}$

but at the same time radiates away energy at a rate P

The maximum energy problem: the role of radiative losses



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who wins?

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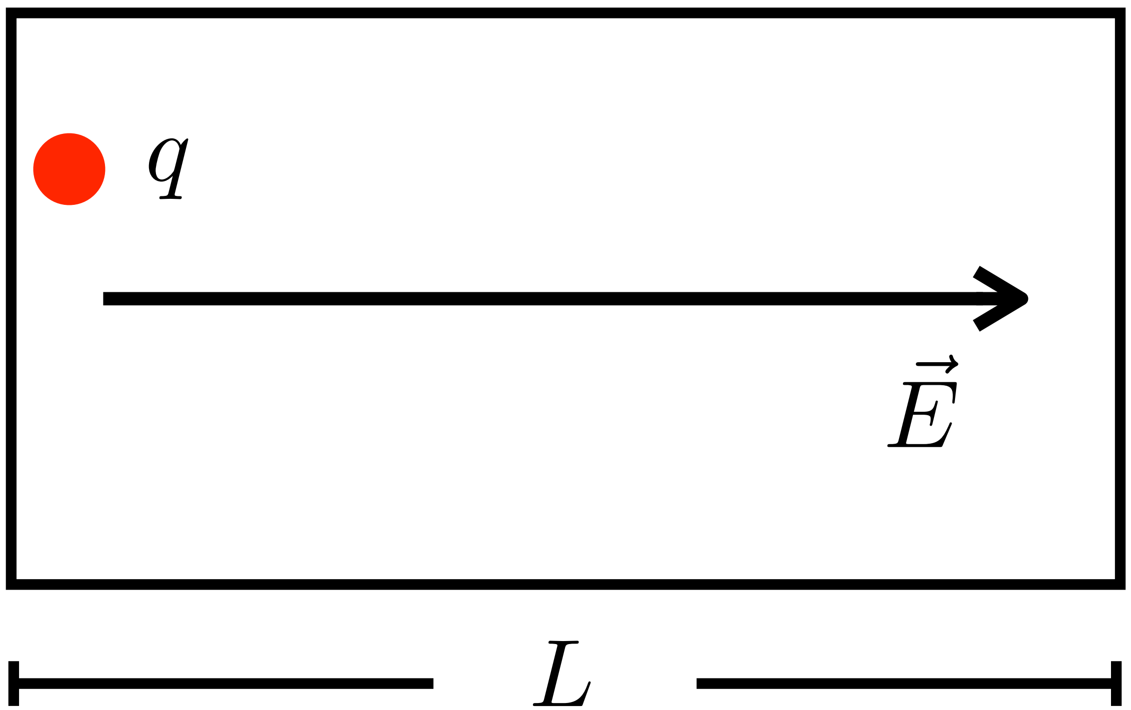

$$\boxed{\frac{P}{\mathrm{d}E_t/\mathrm{d}t}}$$

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The maximum energy problem: the role of radiative losses

loss

$$\frac{P}{\mathrm{d}E_t/\mathrm{d}t} = \frac{2}{3}\frac{q^3 E}{m^2 c^3}\frac{1}{u} \xrightarrow[u=c]{} \frac{2}{3}\frac{q^3 E}{m^2 c^4}$$

gain

does NOT depend on particle energy!

condition to have acceleration —>

$$\frac{2}{3}\frac{q^3 E}{m^2 c^4} < 1$$

or

$$E < \frac{3}{2}\frac{m^2 c^4}{q^3}$$

The maximum energy problem: the role of radiative losses

is this limit really constraining?

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remember: $$\frac{\mathrm{d}E_t}{\mathrm{d}x} = qE$$

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for protons

remember: $$\frac{\mathrm{d}E_t}{\mathrm{d}x} = qE < \frac{3}{2}\left(\frac{mc^2}{q}\right)^2 \approx 10^{25}\mathrm{eV/cm}$$

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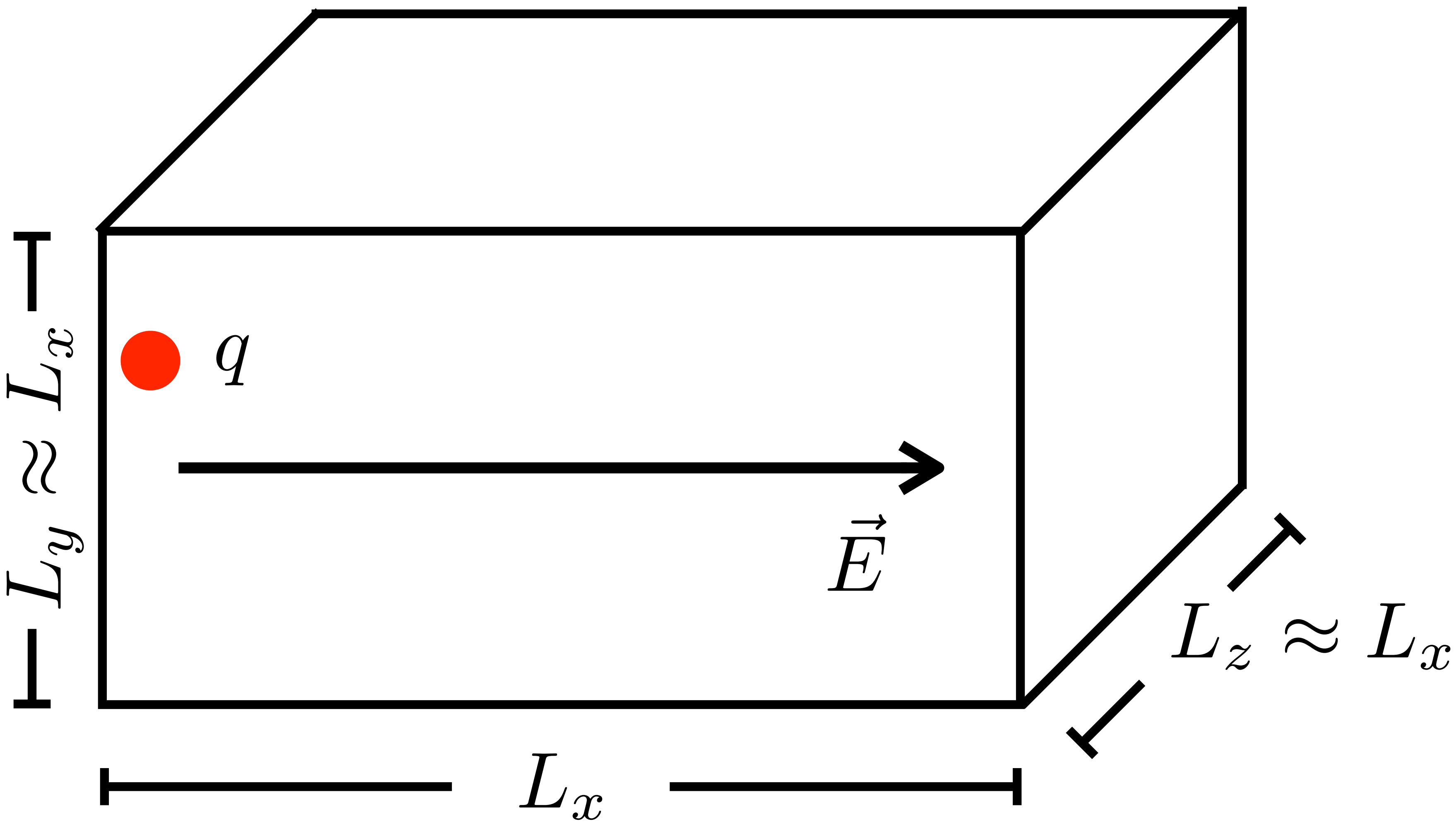
losses are not a problem!

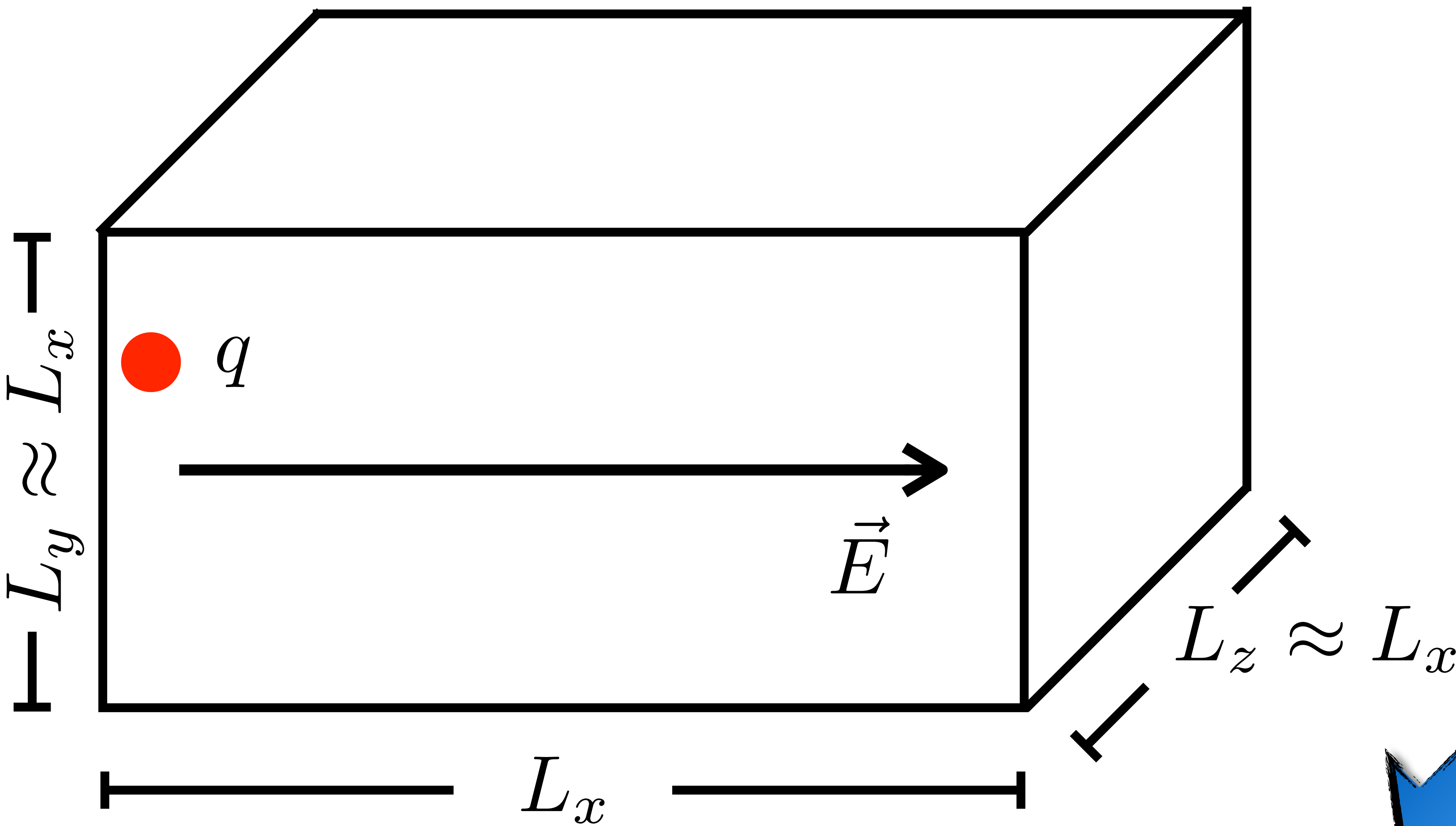
Energetics



Electromagnetic energy in the accelerator:

$$W_{tot} \approx E^2 L^3$$

Energetics



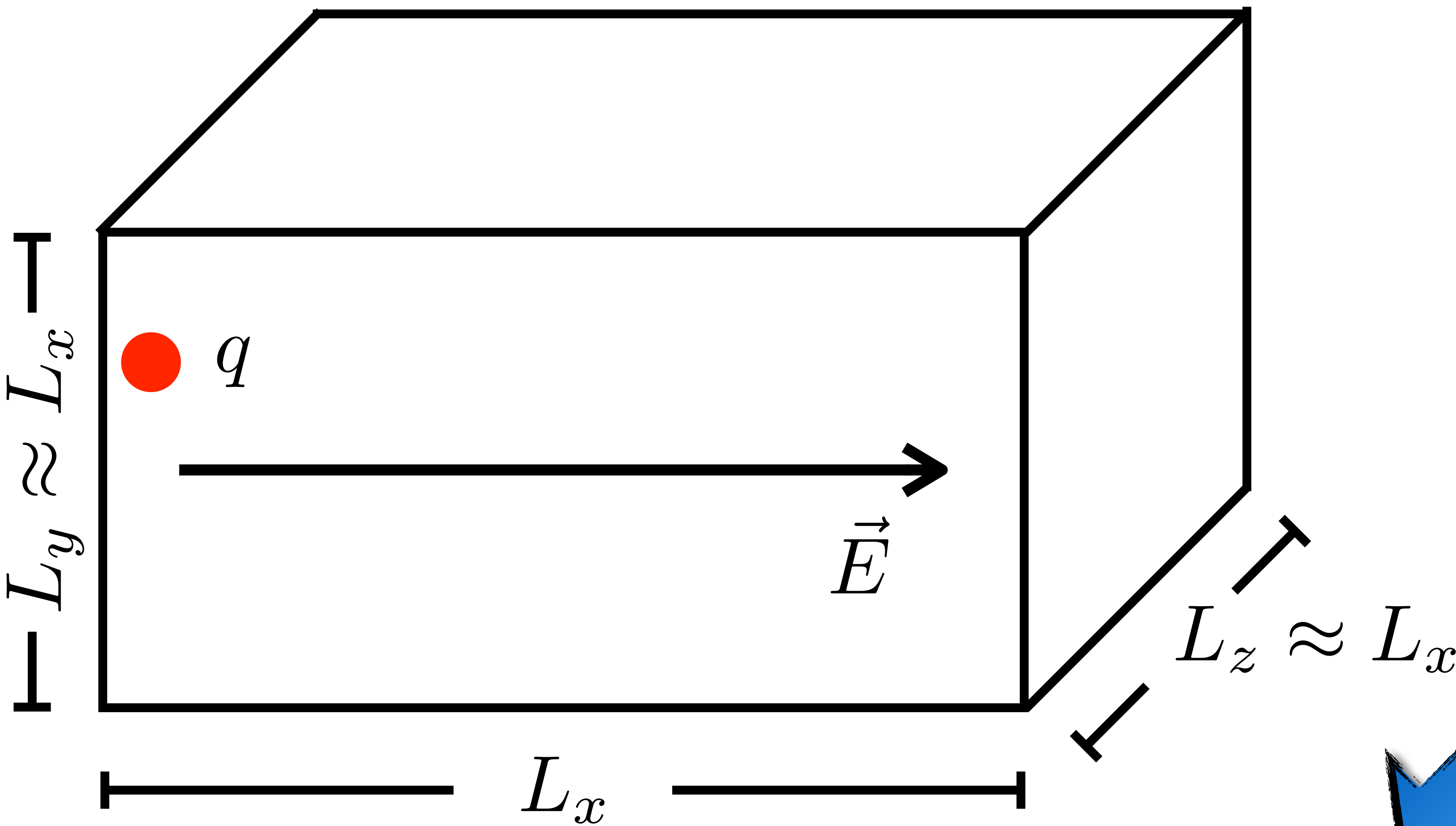
$$E_t^{max} = qEL$$

$$E = \frac{E_t^{max}}{qL}$$

Electromagnetic energy in the accelerator:

$$W_{tot} \approx E^2 L^3 = \left(\frac{E_t^{max}}{q}\right)^2 L \sim 3 \times 10^{53} \left(\frac{E_t^{max}}{10^{20}\mathrm{eV}}\right)^2 \left(\frac{L}{\mathrm{pc}}\right) \ \mathrm{erg}$$

Energetics



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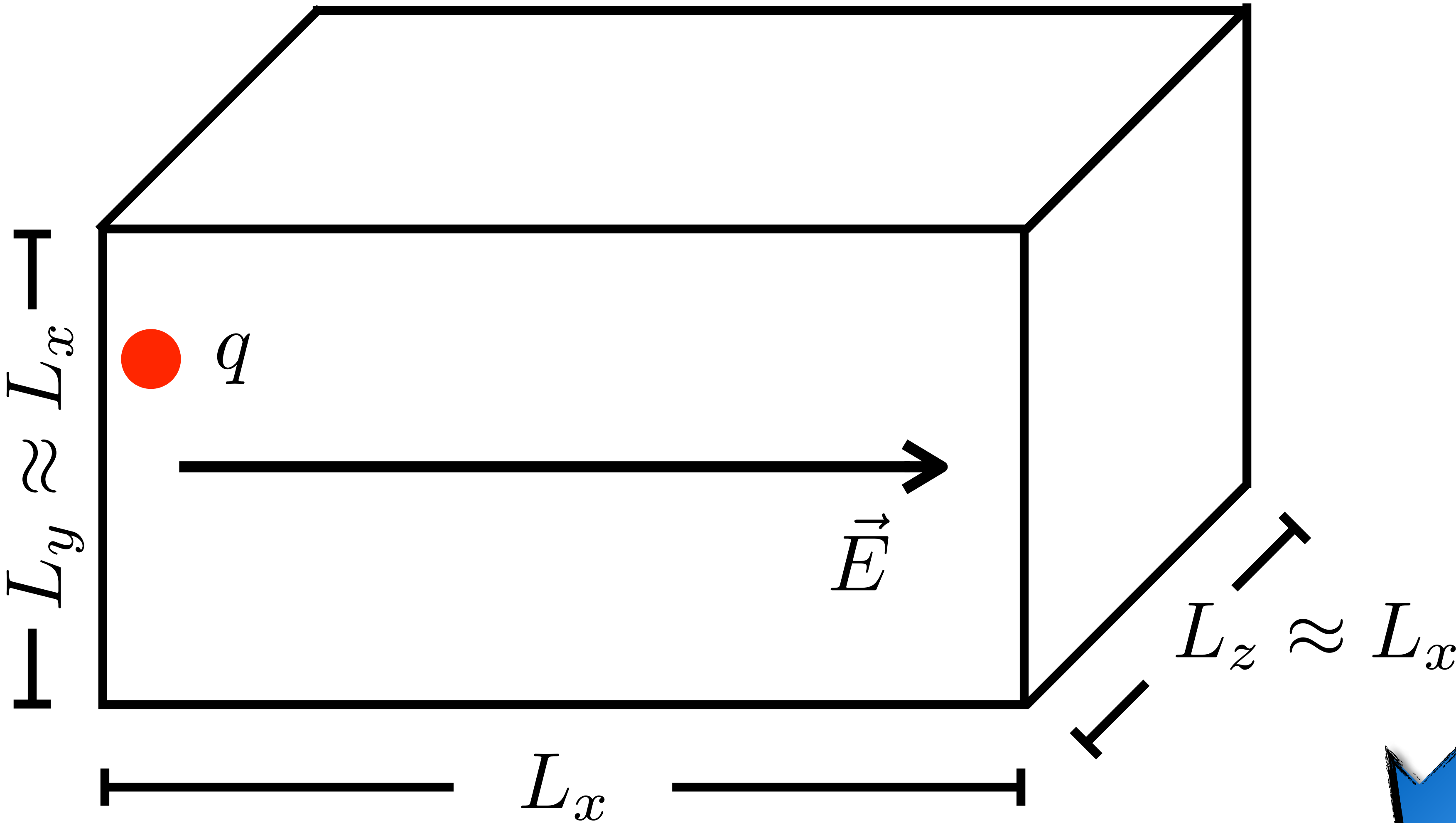
To accelerate to extreme energy we need extremely energetic accelerators!

Energetics



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Electromagnetic energy in the accelerator:

~300 supernovae

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To accelerate to extreme energy we need extremely energetic accelerators!

Things to remember

The simplest (electrostatic) accelerator

- Most effective for:
  - large size
  - strong field
  - large charge
- Radiative losses might prevent acceleration but in fact for any practical purpose can be neglected!
- —> electrostatic accelerators are very efficient!
- To reach larger particle energies accelerators must contain huge amounts electromagnetic energy

What's next?

Is the simplest accelerator a good one? How common are electrostatic accelerators in astrophysical environments?

- Note that an implicit assumption done so far is that acceleration happens in vacuum
- In fact, the Universe is filled with plasma, and this has to be taken into account
- We'll need to study some properties of astrophysical plasmas

Can we keep a static and uniform electric field in an astrophysical plasma?

unfortunately, that's quite difficult…

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An excess of electrical charge is needed to maintain a static electric field. However we should remember…

> "…a basic property of plasma, its tendency towards electrical neutrality. If over a large volume the number of electrons per cubic centimeter deviates appreciably from the corresponding number of positive ions, the electrostatic forces resulting yield a potential energy per particle that is enormously greater than the mean thermal energy. Unless very special mechanisms are involved to support such large potentials, the charged particles will rapidly move in such a way as to reduce these potential difference, i.e., to restore electrical neutrality."

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So, the answer is no...

...but there is still maybe some hope?

Electrostatic accelerators in astrophysical environments?

In most astrophysical environments static electric fields cannot be maintained because of the very high electrical conductivity of plasmas

We need to find ways to energise particles different from the electrostatic accelerator

…or the very special mechanisms invoked by Spitzer

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Most of the concept discussed/developed during our first lecture remain valid

The problem of particle acceleration in astrophysical plasmas

Way-out #1: time varying B

We DO need electric fields to accelerate particles!

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Maxwell equations

$$\nabla \vec{E} = 4\pi \varrho$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \vec{B} = 0$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

Way-out #1: time varying B

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$$\nabla \vec{E} = 4\pi \varrho \ = 0$$ —> plasma quasi-neutrality

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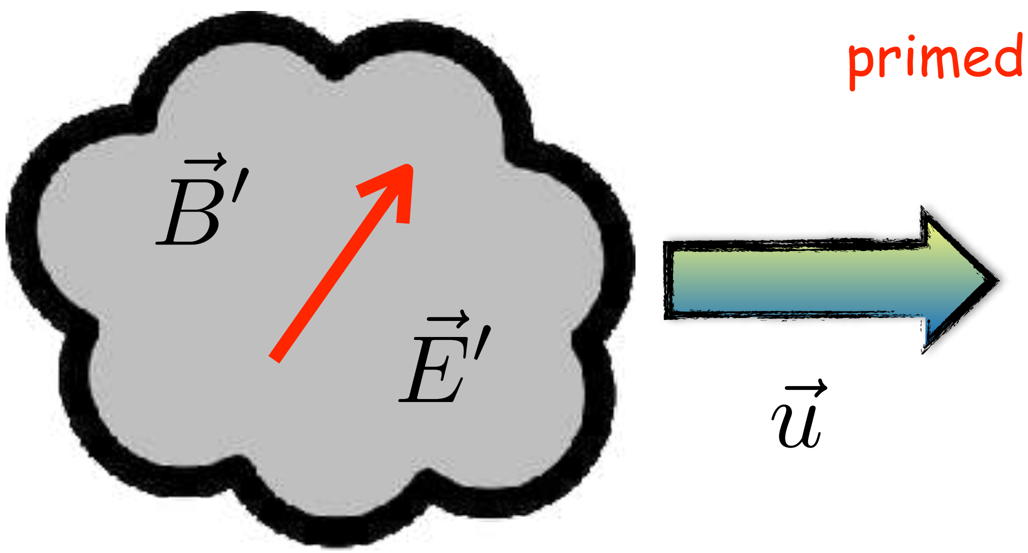
A time varying magnetic field acts as a source of electric field!

Way-out #2: change rest frame

Consider a magnetised cloud of plasma moving at a (non relativistic) velocity u



primed quantities —> cloud frame

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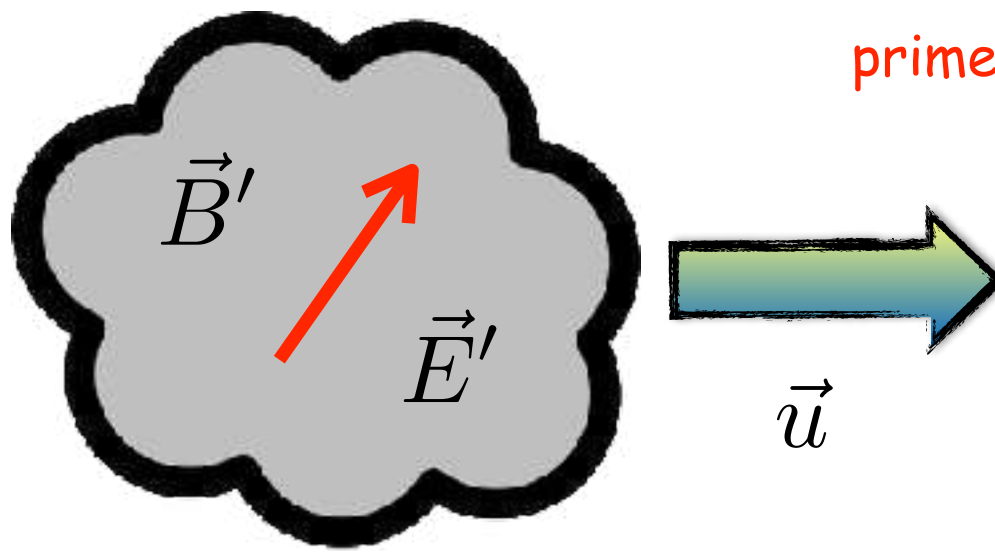

Lorentz transformation

$$\vec{E}' = \vec{E} + \frac{\vec{u}}{c} \times \vec{B}$$

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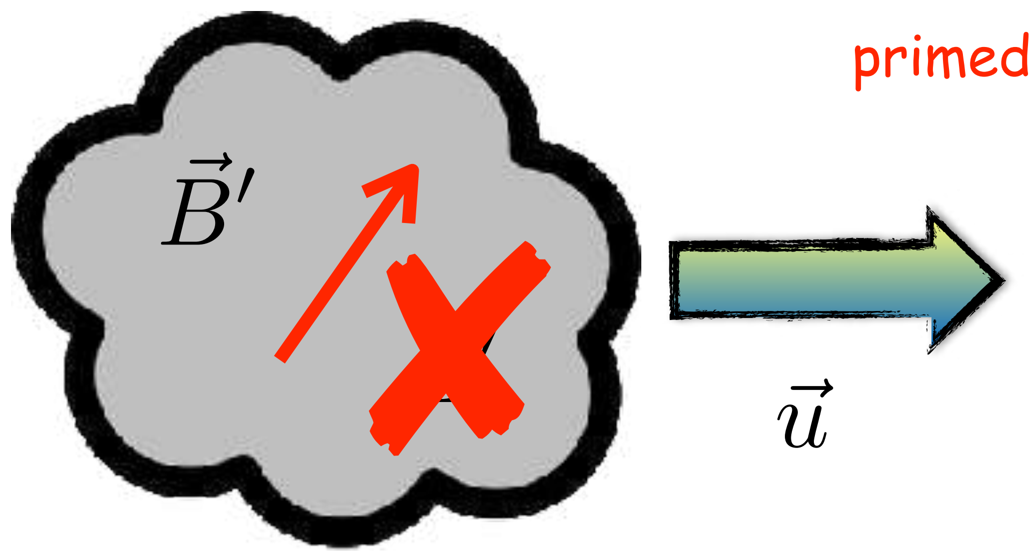

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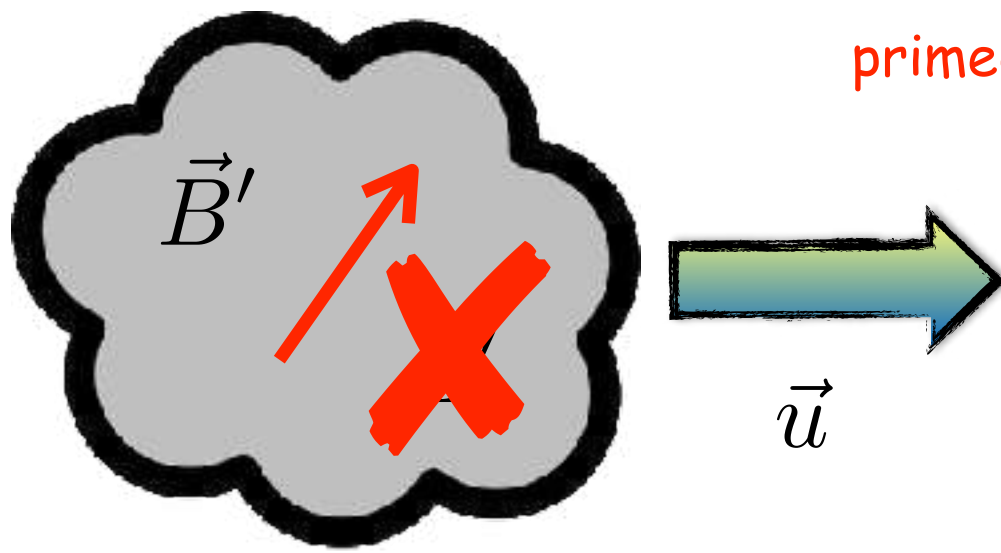

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an observer in the lab frame sees an electric field!

Order of magnitude estimates of the induced electric field

time-varying B-field

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

Order of magnitude estimates of the induced electric field

time-varying B-field

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characteristic length

$$\nabla \times \quad \rightarrow \quad \frac{1}{L}$$

$$\frac{\partial}{\partial t} \quad \rightarrow \quad \frac{1}{T}$$

characteristic time

Order of magnitude estimates of the induced electric field

time-varying B-field

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characteristic length

$$\nabla \times \quad \rightarrow \quad \frac{1}{L}$$

$$\frac{\partial}{\partial t} \quad \rightarrow \quad \frac{1}{T}$$

characteristic time

characteristic velocity

$$E \approx \frac{L}{T}\frac{B}{c} \approx \frac{U}{c}B$$

Order of magnitude estimates of the induced electric field

time-varying B-field

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

characteristic length

$$\nabla \times \quad \rightarrow \quad \frac{1}{L}$$

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$$E \approx \frac{L}{T}\frac{B}{c} \approx \frac{U}{c}B$$

characteristic velocity

Lorentz transformation

$$\vec{E} = -\frac{\vec{u}}{c} \times \vec{B}$$

$$E \approx \frac{U}{c}B$$

Hillas criterium

Let's go back to the results obtained for the electrostatic accelerator

$$E_t^{max} = qEL$$

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$$E_t^{max} = qEL$$

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$$\Longrightarrow$$

electric charge

velocity

$$E_t^{max} \approx \left(\frac{q}{c}\right) B \; U \; L$$

B-field

size

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B-field

size

$$\boxed{E_t^{max} \approx 3 \times 10^{12} Z \left(\frac{B}{\mu\mathrm{G}}\right) \left(\frac{U}{1000\ \mathrm{km/s}}\right) \left(\frac{L}{\mathrm{pc}}\right) \ \mathrm{eV}}$$

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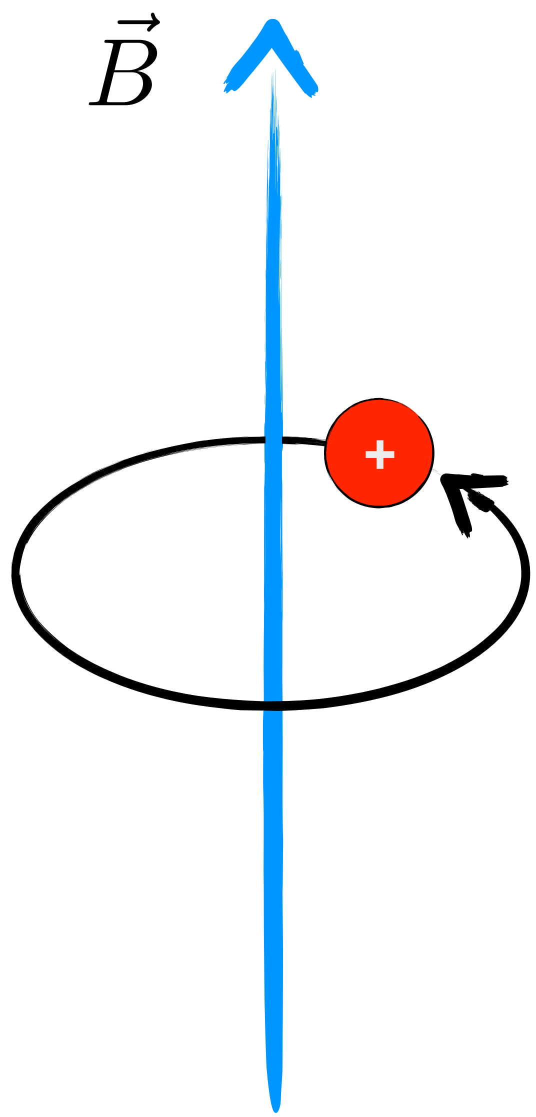
very general, we didn't assume anything about the nature of the accelerator!

Hillas criterium

Important result: we don't need to know how particles are accelerated in order to know whether or not a given astrophysical object is, potentially, a good particle accelerator!

Hillas criterium

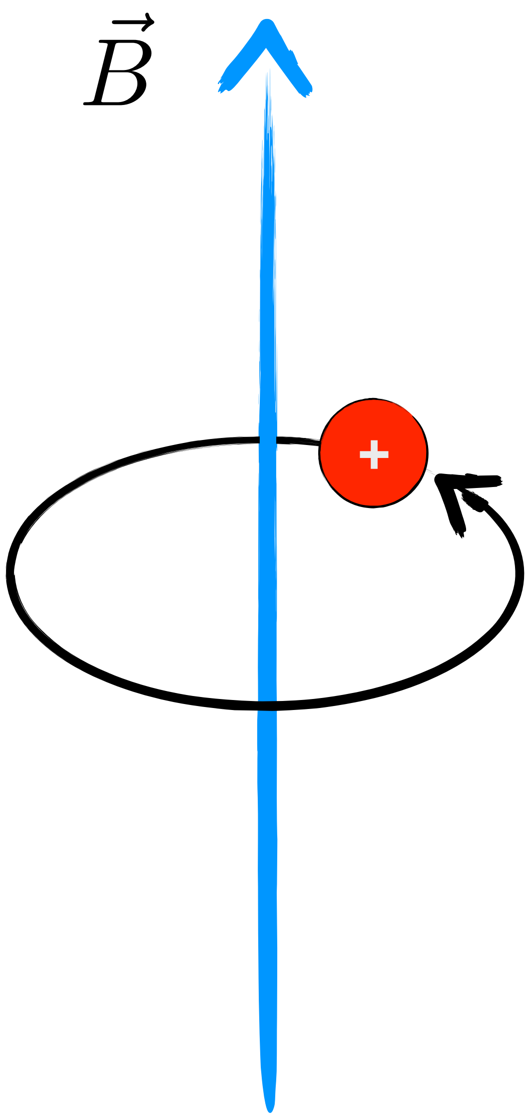


Larmor radius

$$R_L = \frac{E_t^{max}}{qB}$$

Hillas, Ann. Rev. Astron. Astrophys. (1984)

Hillas criterium



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in order to get the most optimistic (but still plausible) result —> u/c ~ 1

$$E_t^{max} \approx \left(\frac{q}{c}\right) B\ U\ L \quad \Longrightarrow \quad R_L(E_t^{max}) \approx L$$

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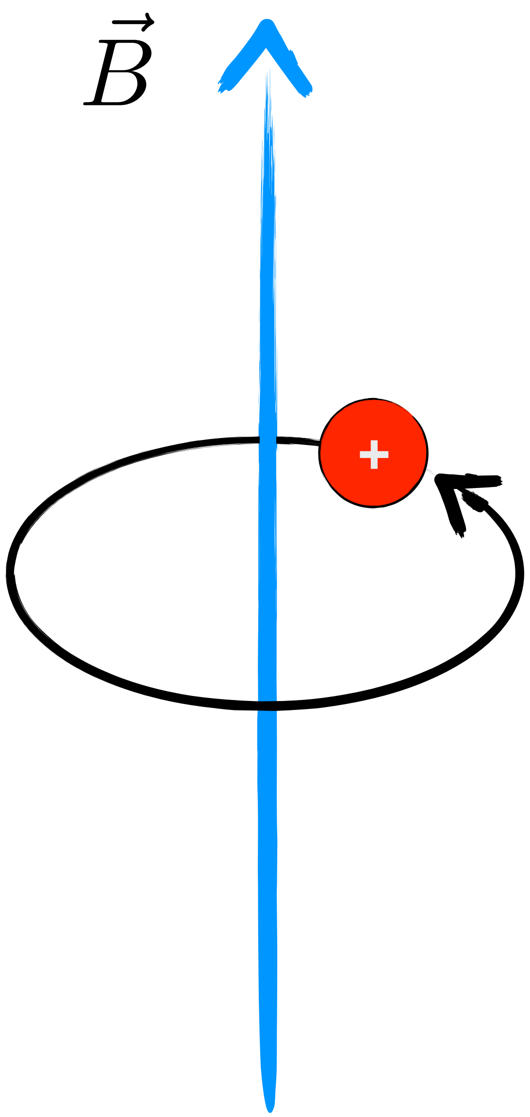

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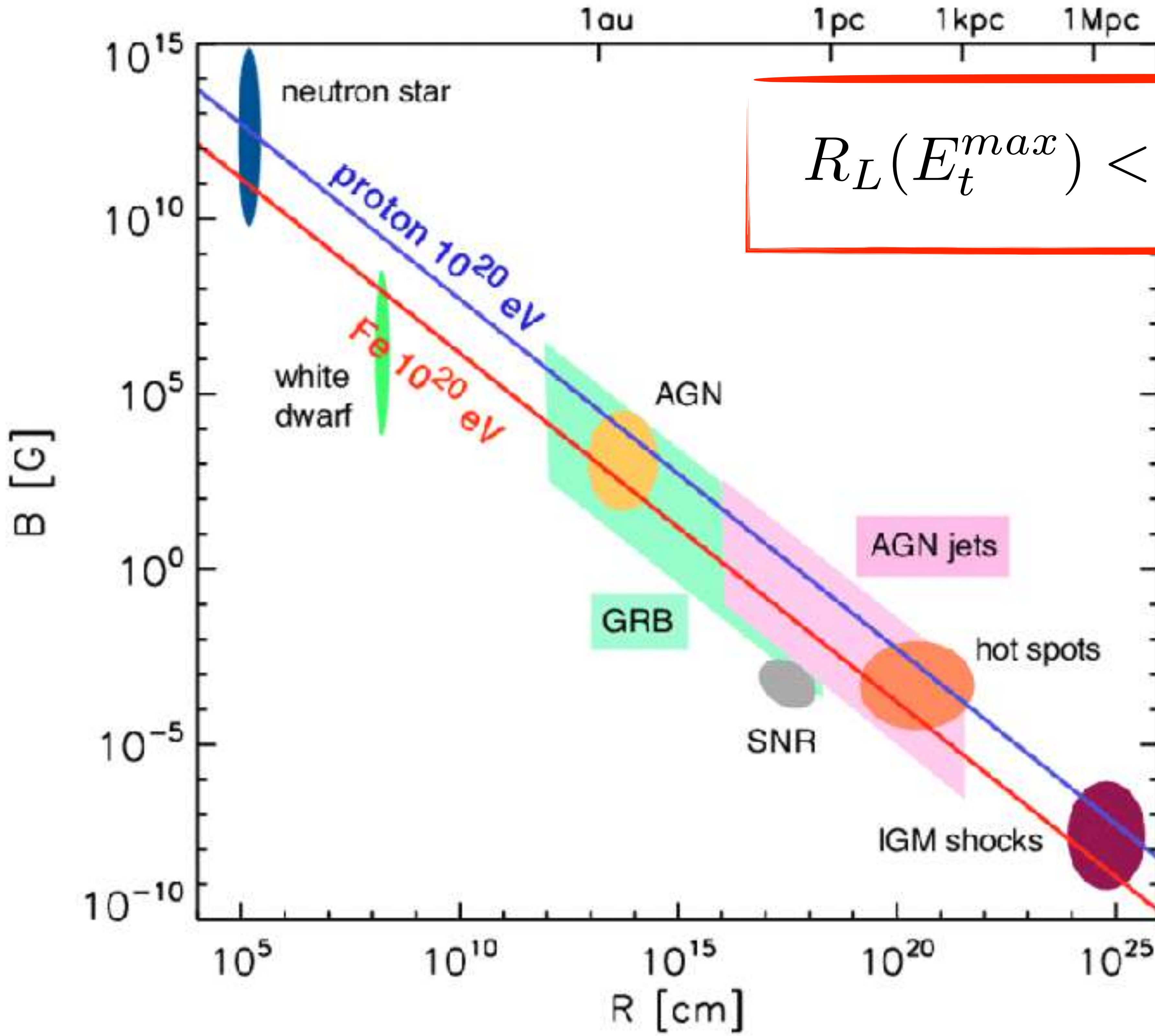
In order to be accelerated, a particle must be confined within its accelerator!

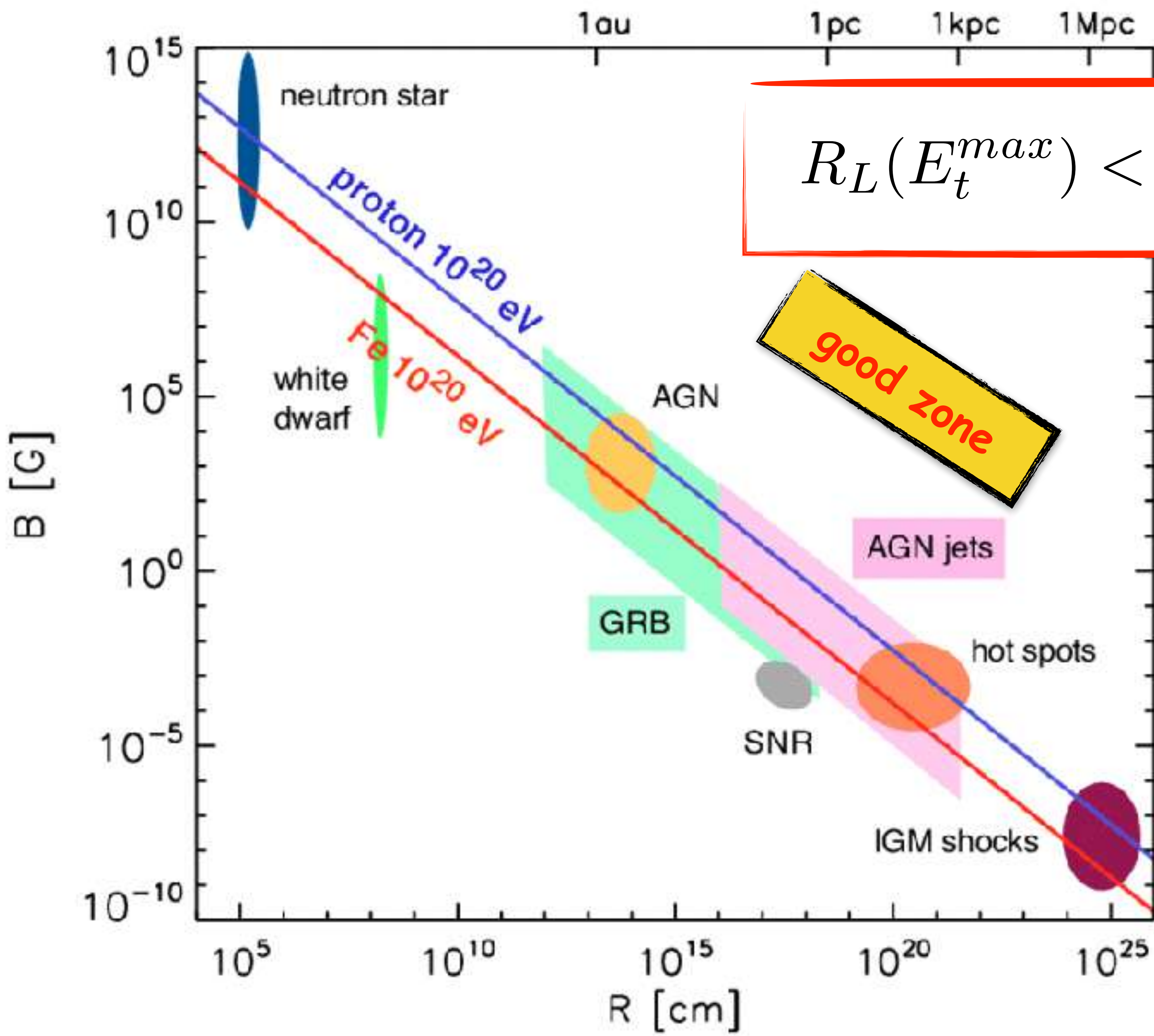
Hillas, Ann. Rev. Astron. Astrophys. (1984)

The Hillas plot



$$R_L(E_t^{max}) < L \longrightarrow LB > E_t^{max}$$

The Hillas plot



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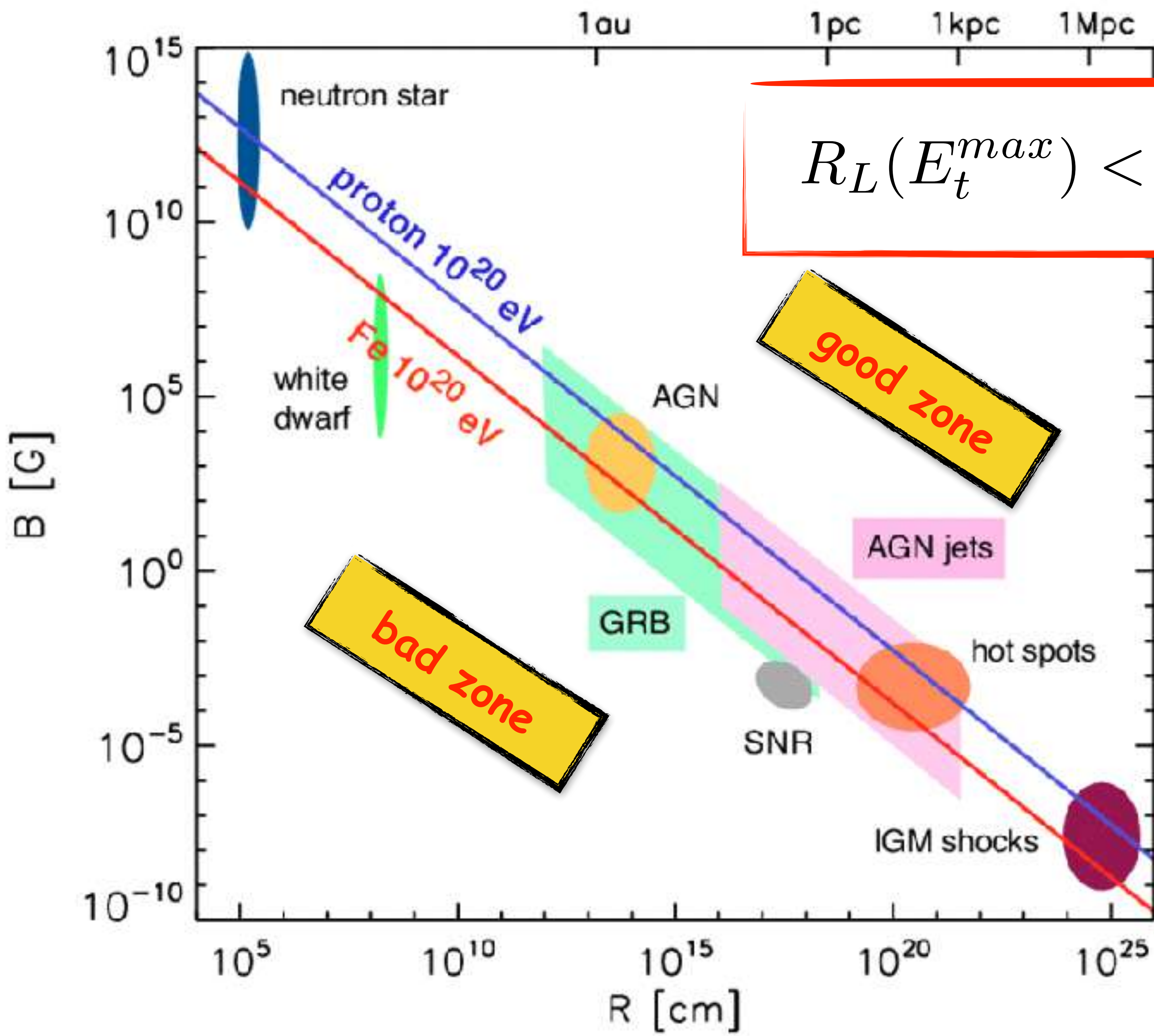

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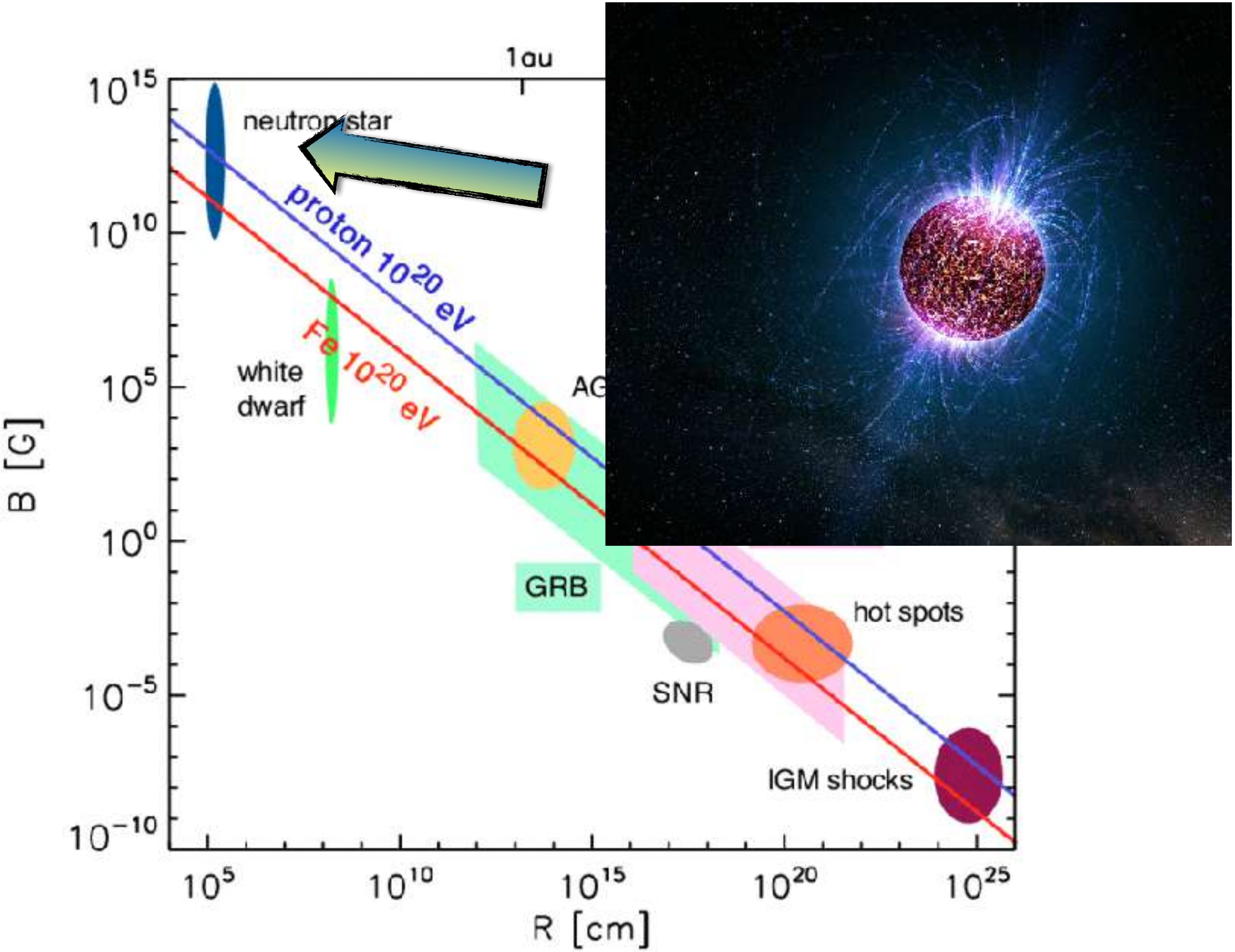
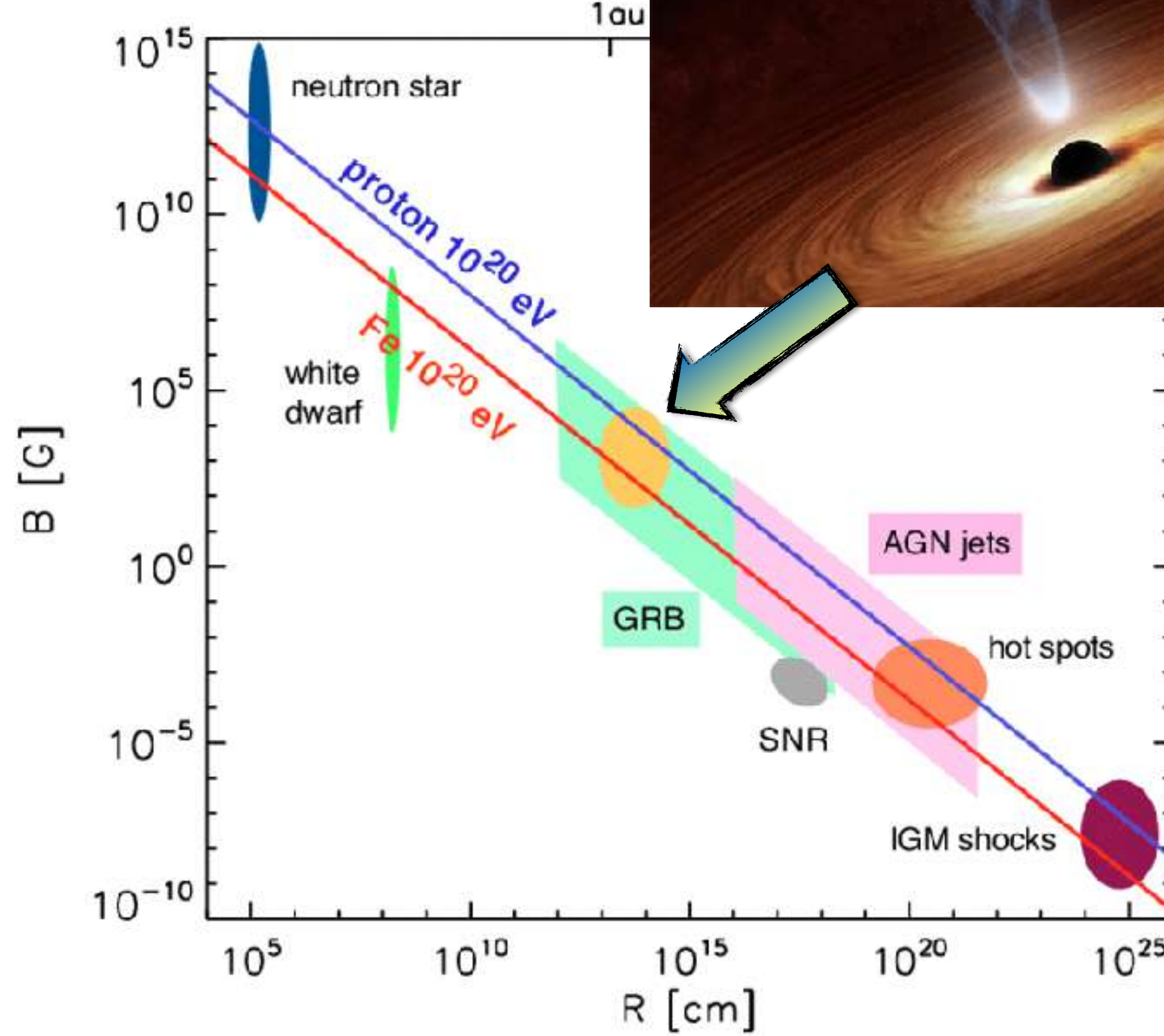


The Hill


1au
neutron star
proton 10^{20} eV
Fe 10^{20} eV
white dwarf
GRB
SNR
AGN jets
hot spots
IGM shocks
B [G]
R [cm]
10^{15}
10^{10}
10^{5}
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10^{-5}
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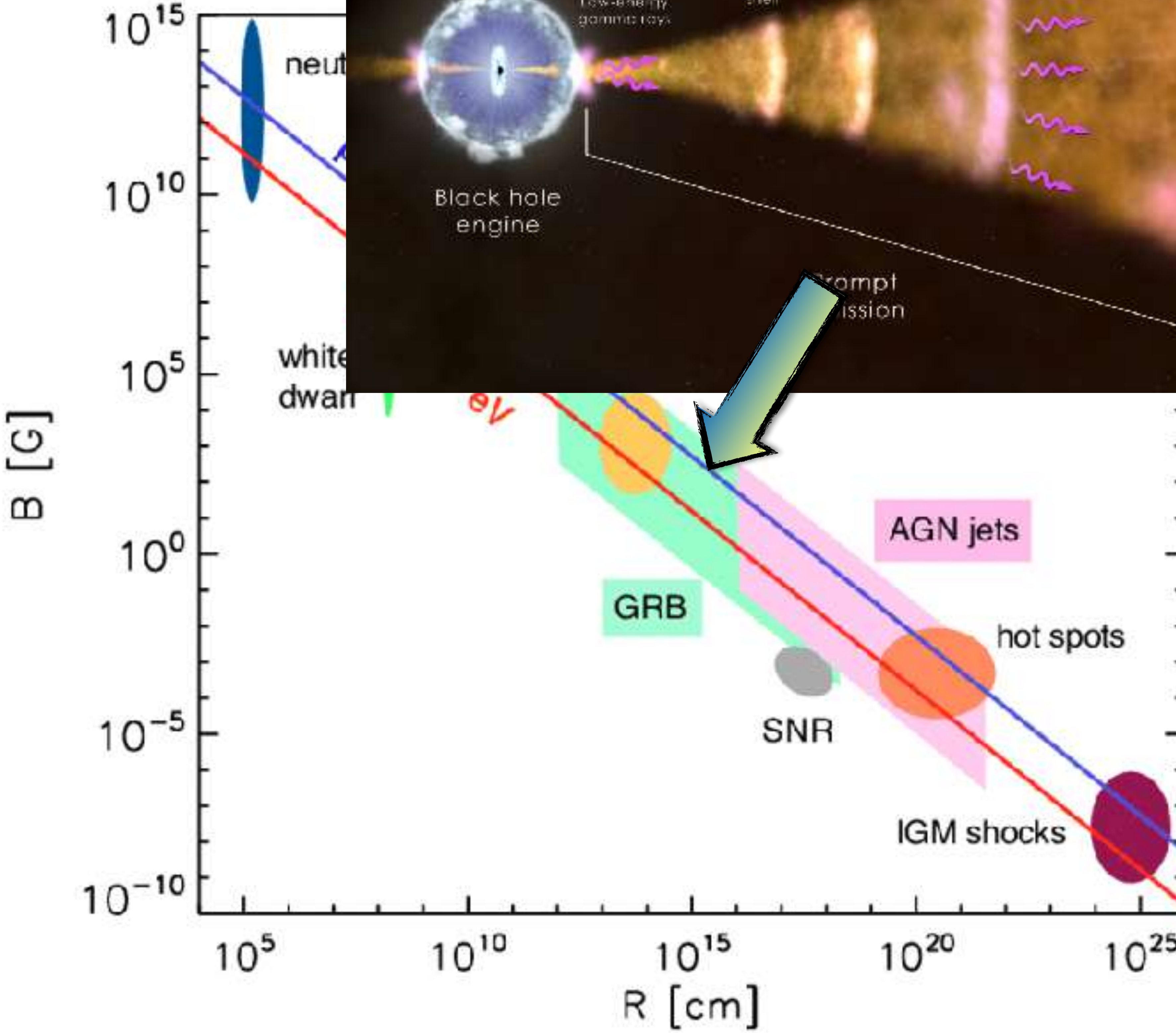
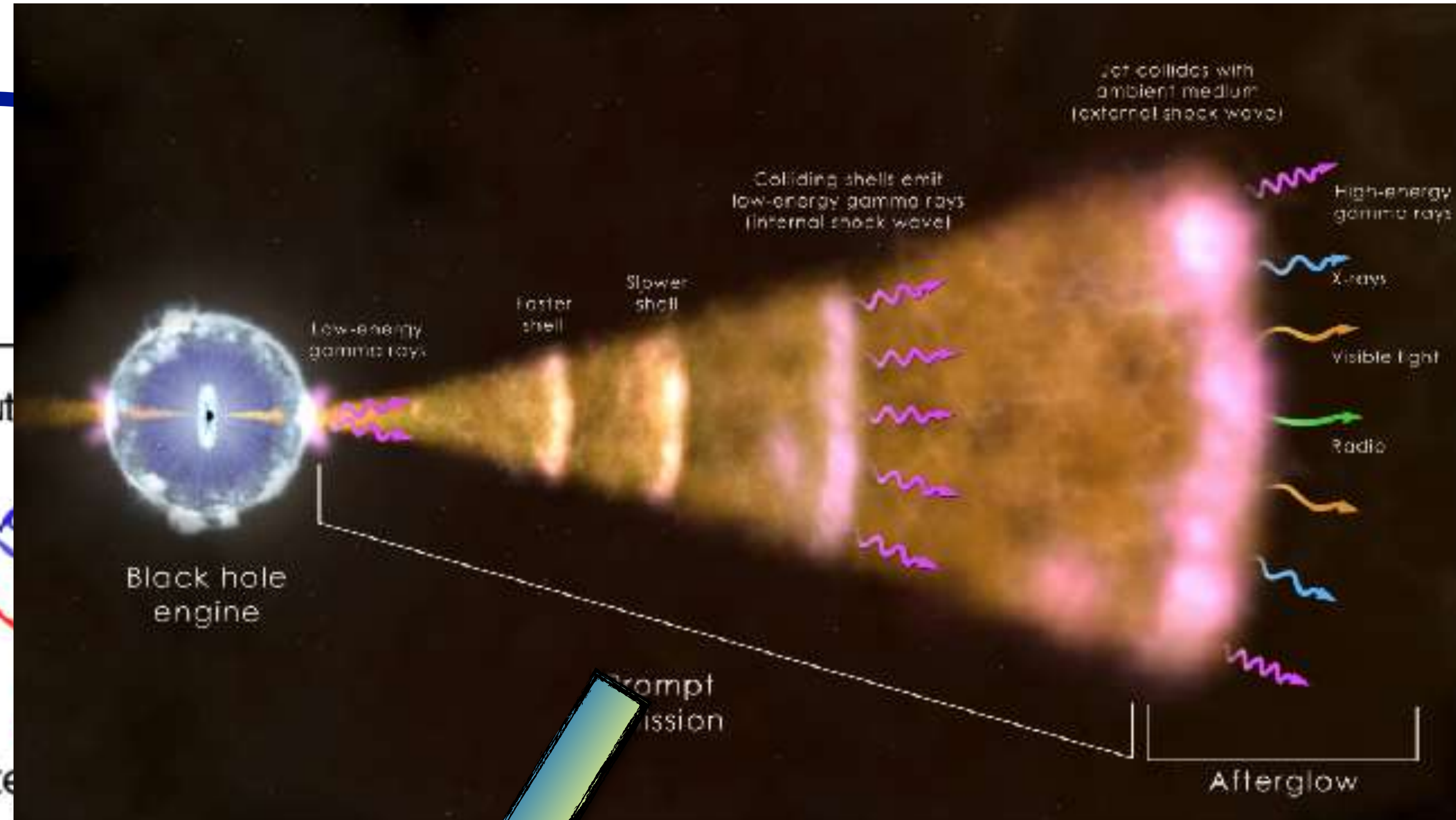
10^{5}
10^{10}
10^{15}
10^{20}
10^{25}

T

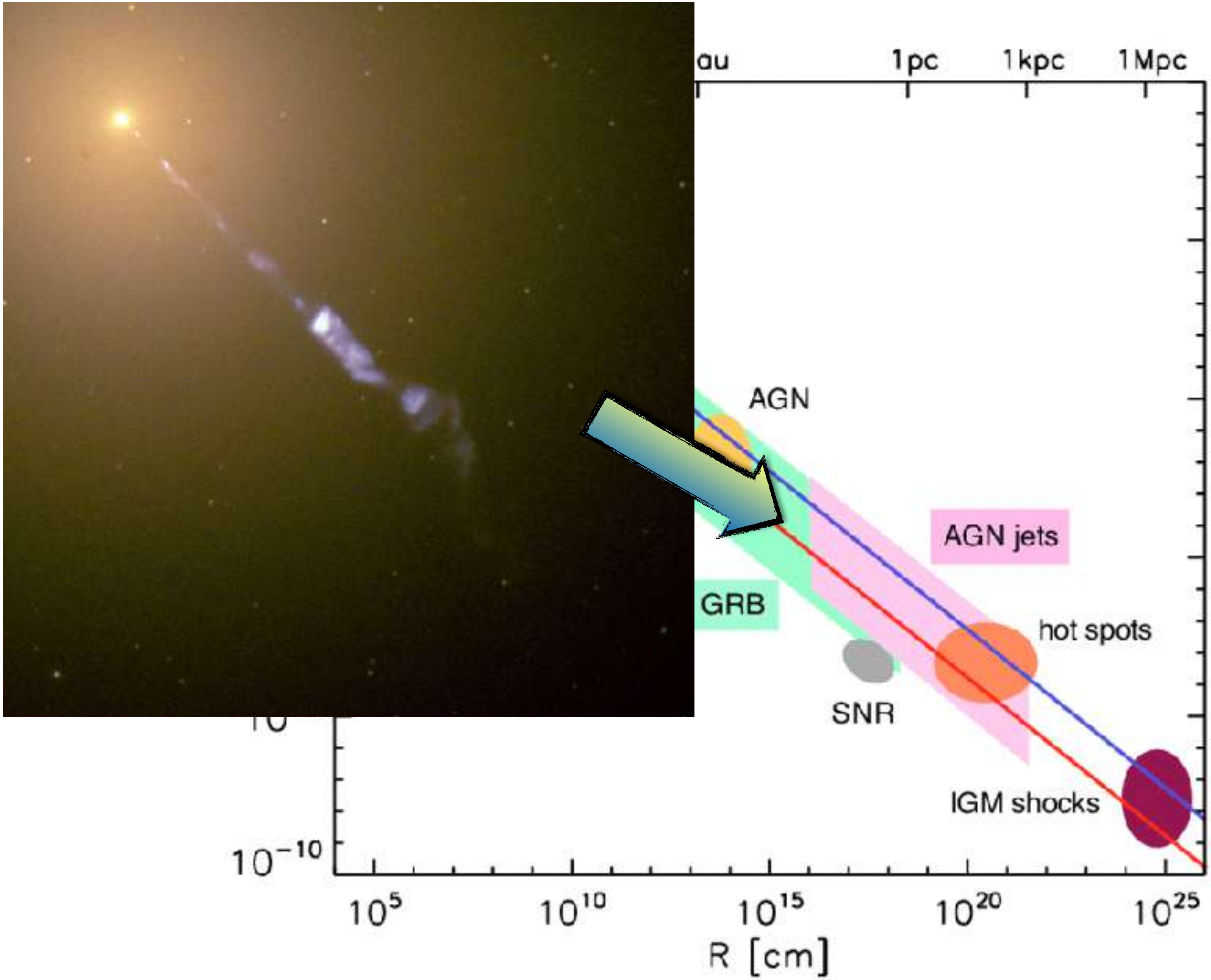


10^{15}
10^{10}
10^{5}
10^{0}
10^{-5}
10^{-10}
B [G]
neut
white
dwarf
eV
AGN jets
GRB
hot spots
SNR
IGM shocks
10^{5}
10^{10}
10^{15}
10^{20}
10^{25}
R [cm]



Jet collides with ambient medium (external shock wave)
Colliding shells emit low-energy gamma rays (internal shock wave)
Faster shell
Slower shell
Low-energy gamma rays
Black hole engine
rompt
ission
Afterglow
High-energy gamma rays
X-rays
Visible light
Radio

The Hillas plot



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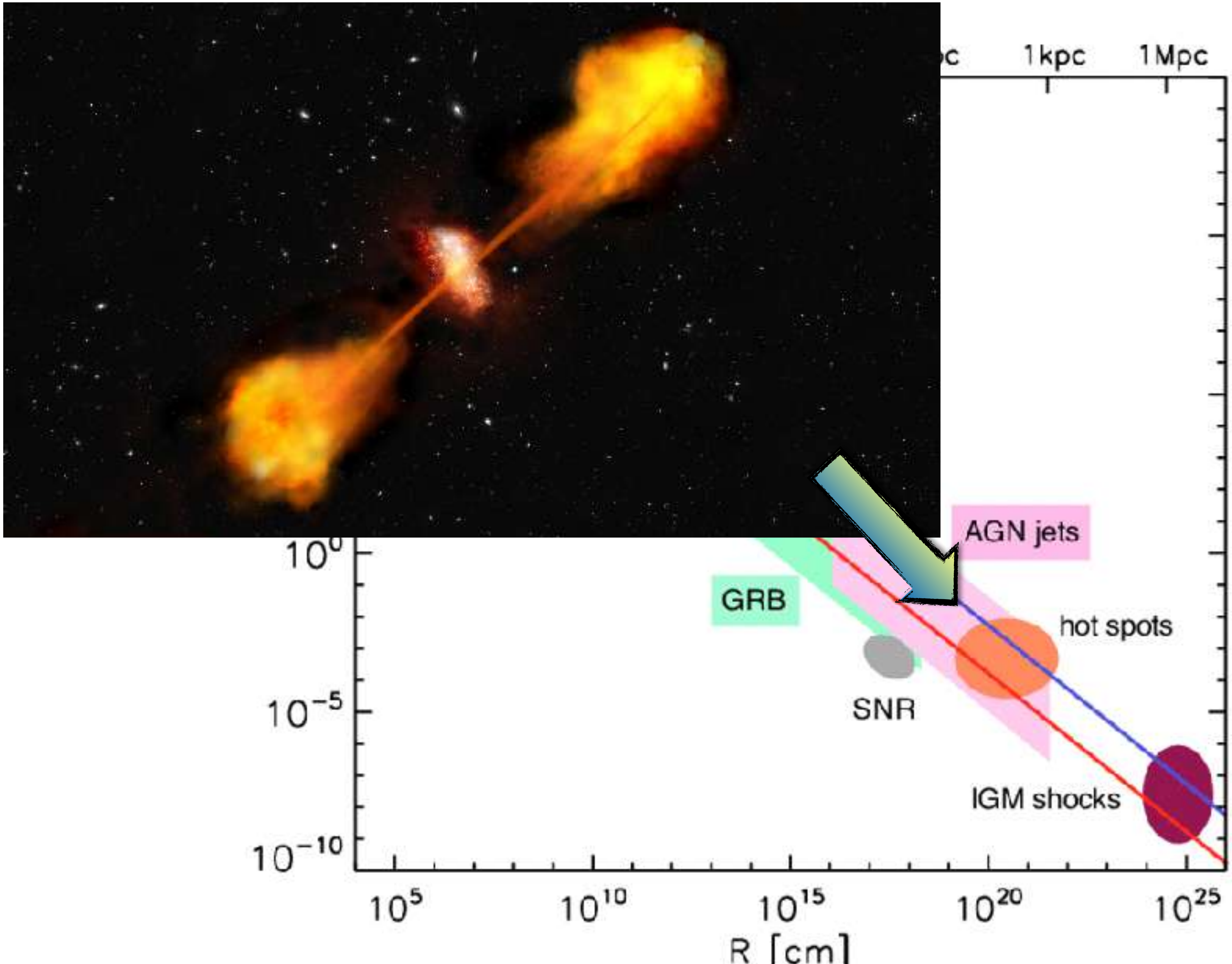

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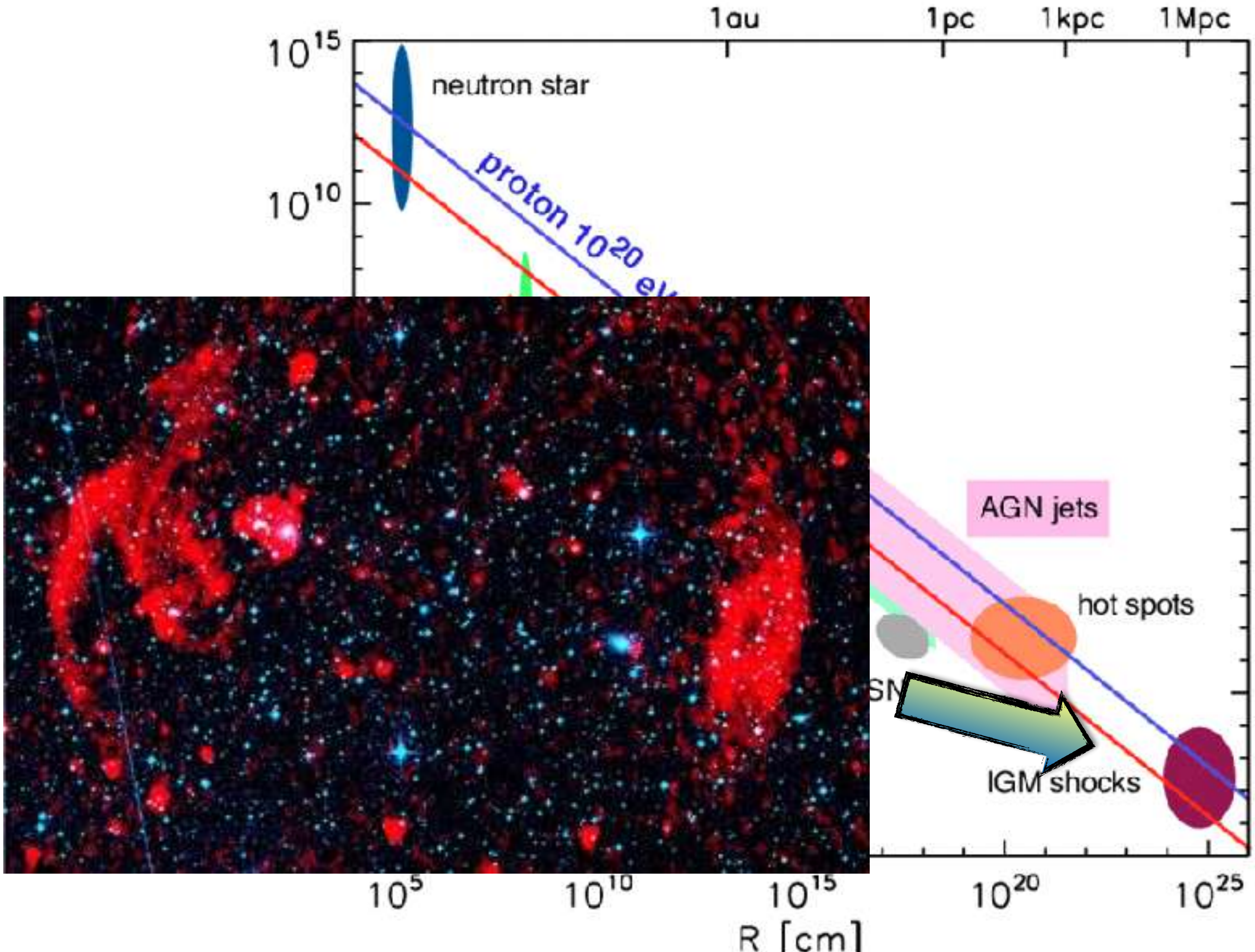

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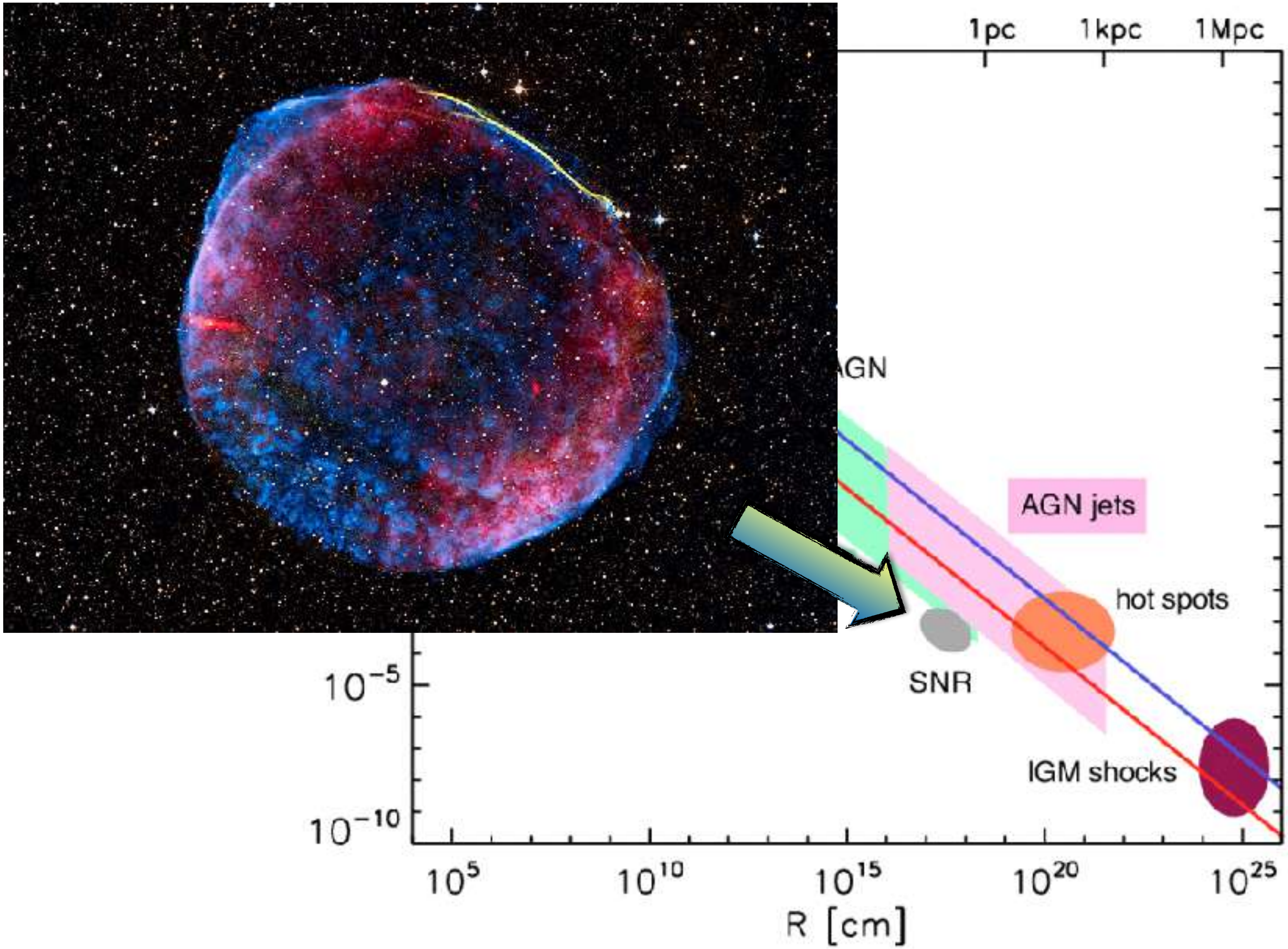

Maximum energy and fastest acceleration rate from the Hillas criterium

Electrostatic accelerator

maximum energy —> $$E_t^{max} = qEL$$

acceleration rate—> $$\frac{\mathrm{d}E_t}{\mathrm{d}x} = qE$$

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$$\frac{\mathrm{d}E_t}{\mathrm{d}x} = qE \quad \mathrm{d}x = c\mathrm{d}t$$

Hillas criterium

$$E_t^{max} = qBL$$

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from now on we can remove the subscript t to distinguish particle energies from electric fields

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Induced electric field

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from now on we can remove the subscript t to distinguish particle energies from electric fields

The Hillas criterium states that particles with energy above Emax cannot be confined within the system and escape!

Energy budget from the Hillas criterium

How large is the total magnetic energy contained in the accelerator?

Energy budget from the Hillas criterium

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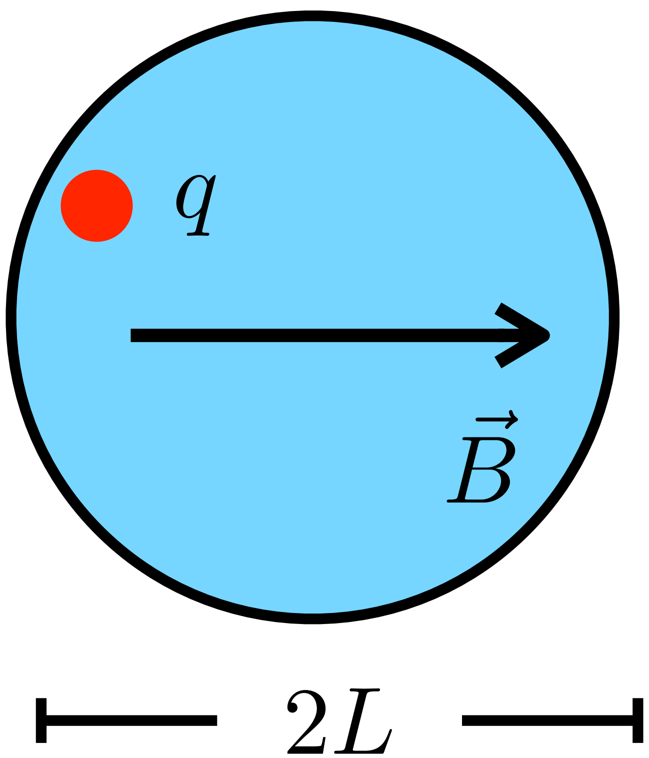

magnetic energy

$$W_B \sim \frac{B^2}{8\pi}\frac{4\pi}{3}L^3$$

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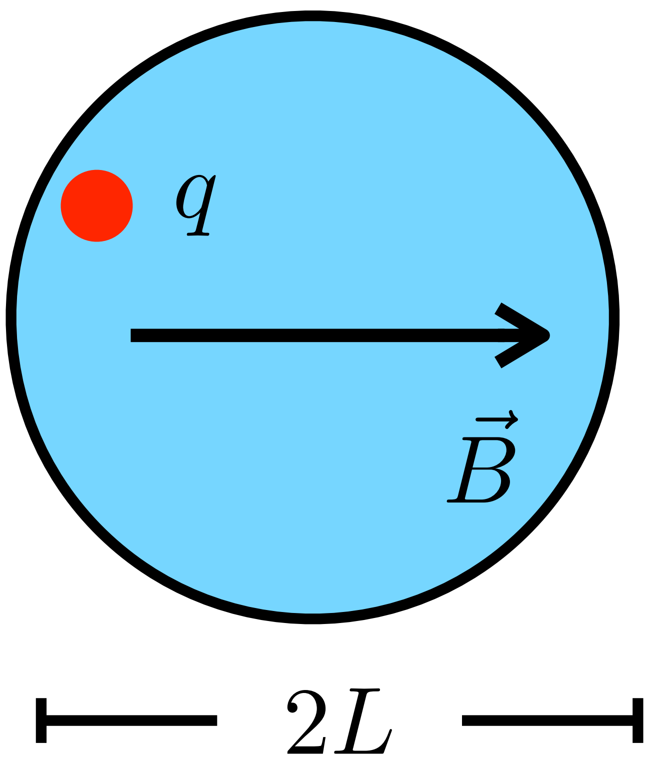

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Hillas criterium

$$R_L(E_{max}) = \frac{E_{max}}{qB} \sim L$$

particle energy

$$B = \frac{E_{max}}{qL} \sim 0.1 \left(\frac{E_{max}}{10^{20}\mathrm{eV}}\right)\left(\frac{L}{\mathrm{pc}}\right)^{-1} \mathrm{G}$$

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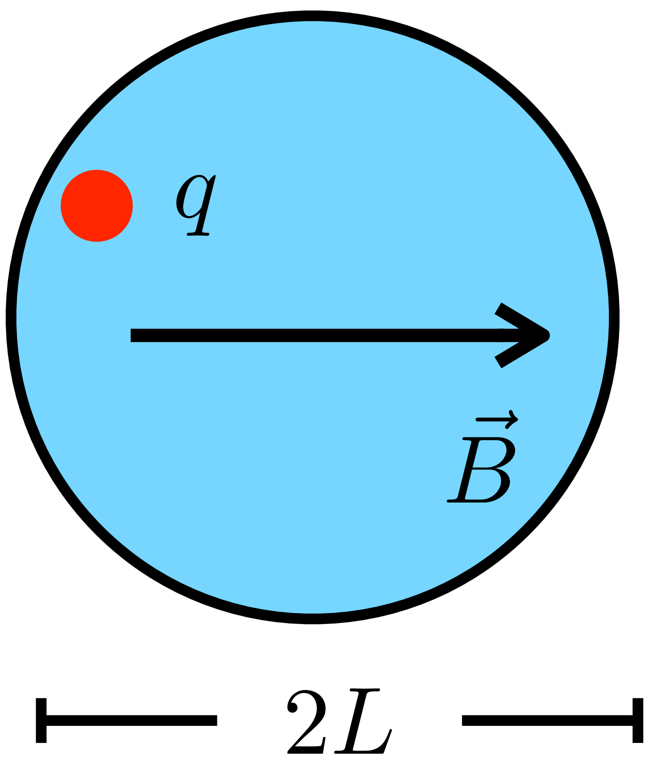

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$$W_B = \frac{E_{max}^2 L}{6\ q^2} \approx 5 \times 10^{52} Z^{-1} \left(\frac{E_{max}}{10^{20}\mathrm{eV}}\right)^2 \left(\frac{L}{\mathrm{pc}}\right) \mathrm{erg}$$

As reference value: 1 supernova releases 10^{51} erg in form of kinetic energy

What about energy losses?

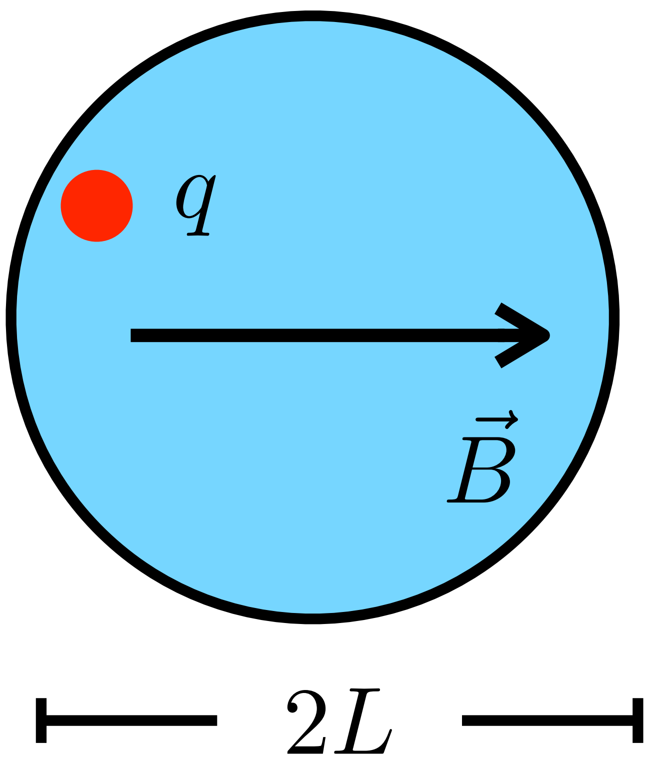
Implicit assumption made in deriving the Hillas criterium: energy losses can be neglected. But this is, in general, not true!

The best accelerator

(allowed by fundamental physics)

Hillas criterium and radiative losses

The Hillas criterium does not consider the possible effects of radiative losses



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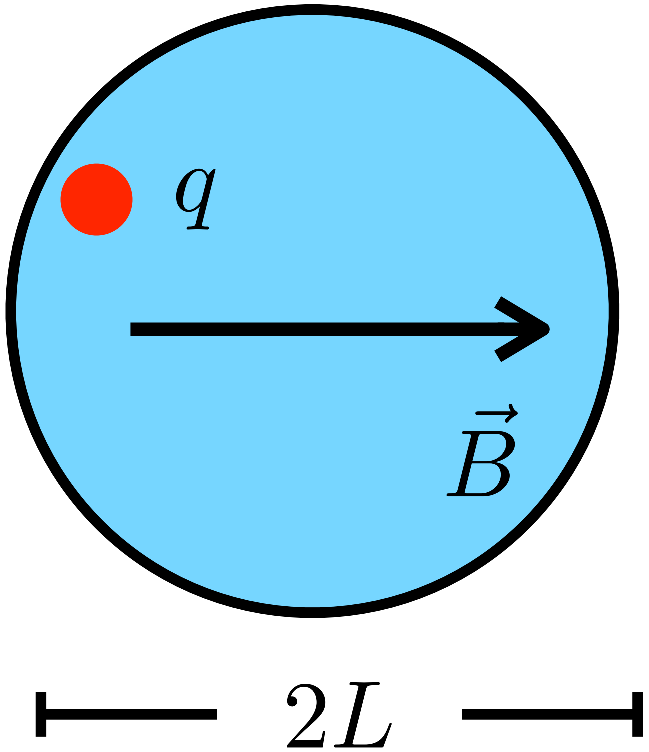

- An electrostatic accelerator would be such a great accelerator because radiative losses can be neglected

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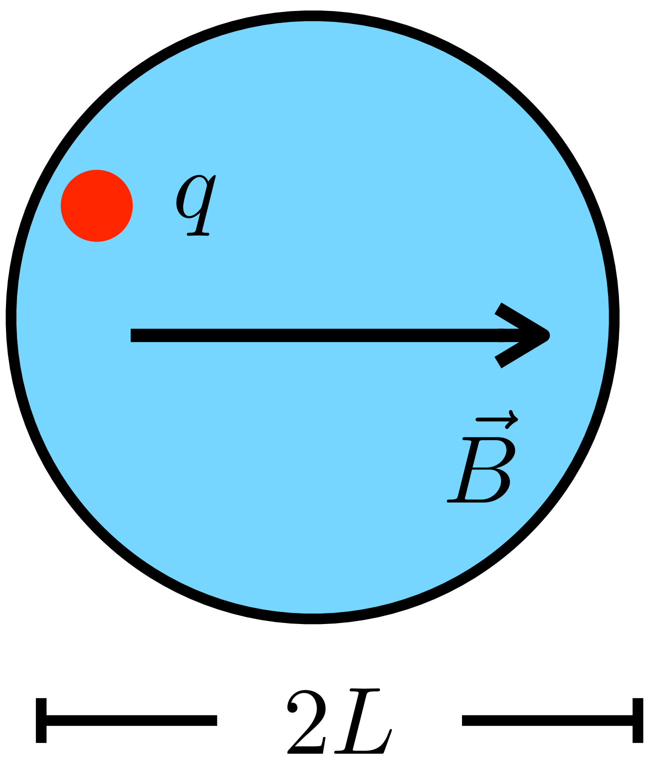

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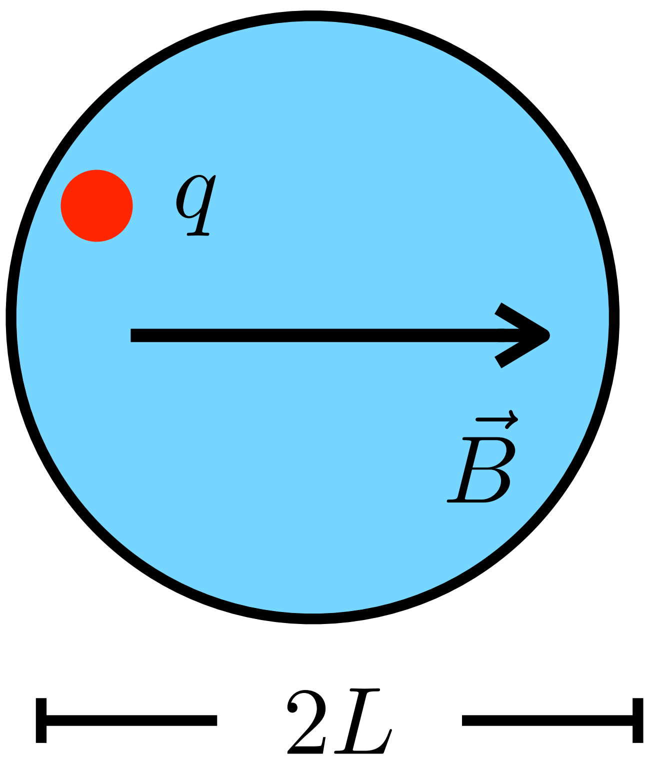

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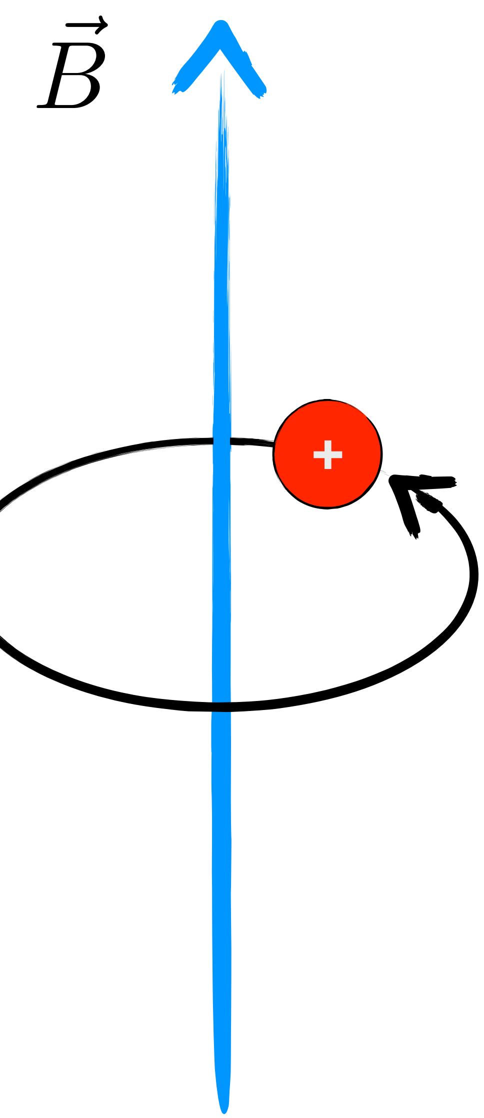

Lorentz force

$$F_L = \frac{q}{c}\vec{v} \times \vec{B}$$

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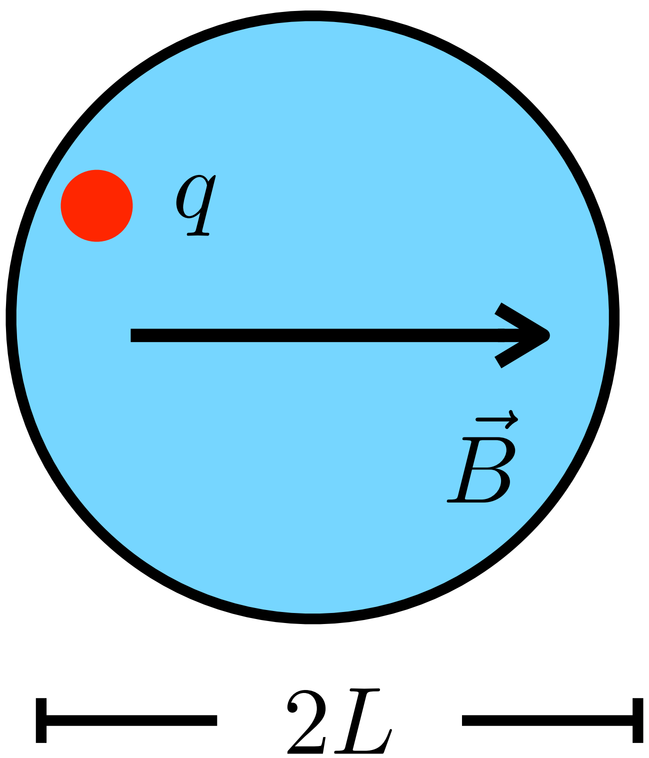

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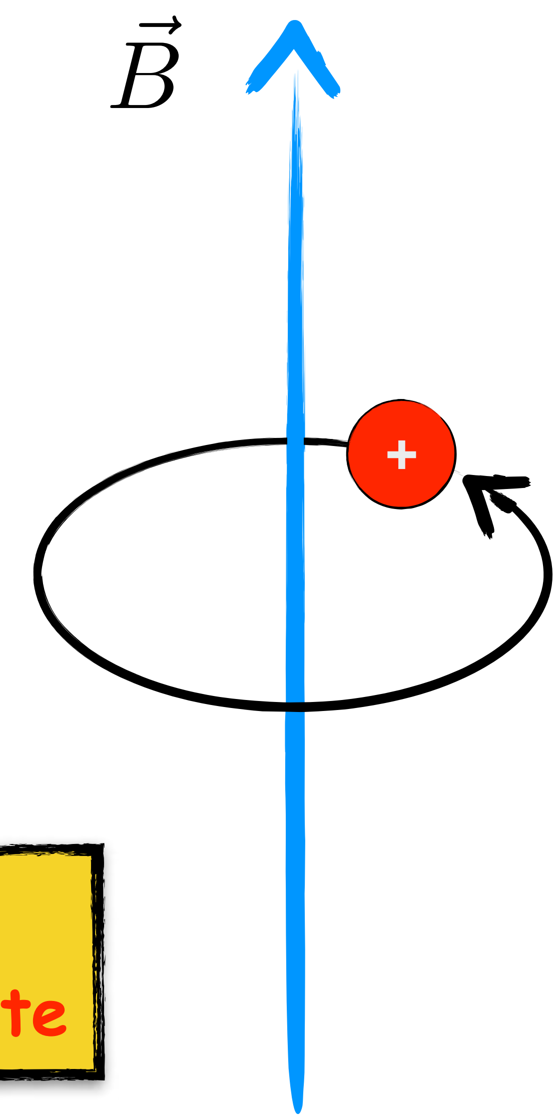

Lorentz force

$$F_L = \frac{q}{c}\vec{v} \times \vec{B} = \gamma m \frac{\mathrm{d}v_\perp}{\mathrm{d}t}$$

acceleration

$$a_\perp = \frac{q v_\perp B}{\gamma m c}$$

accelerated particles radiate

Synchrotron radiation

(for more details see "Radiative processes in astrophysics" Rybicki & Lightman)

Radiated power

$$P = \frac{2q^2}{3c^3}\gamma^4 \left[\gamma^2 a_{\parallel}^2 + a_{\perp}^2\right]$$

acceleration

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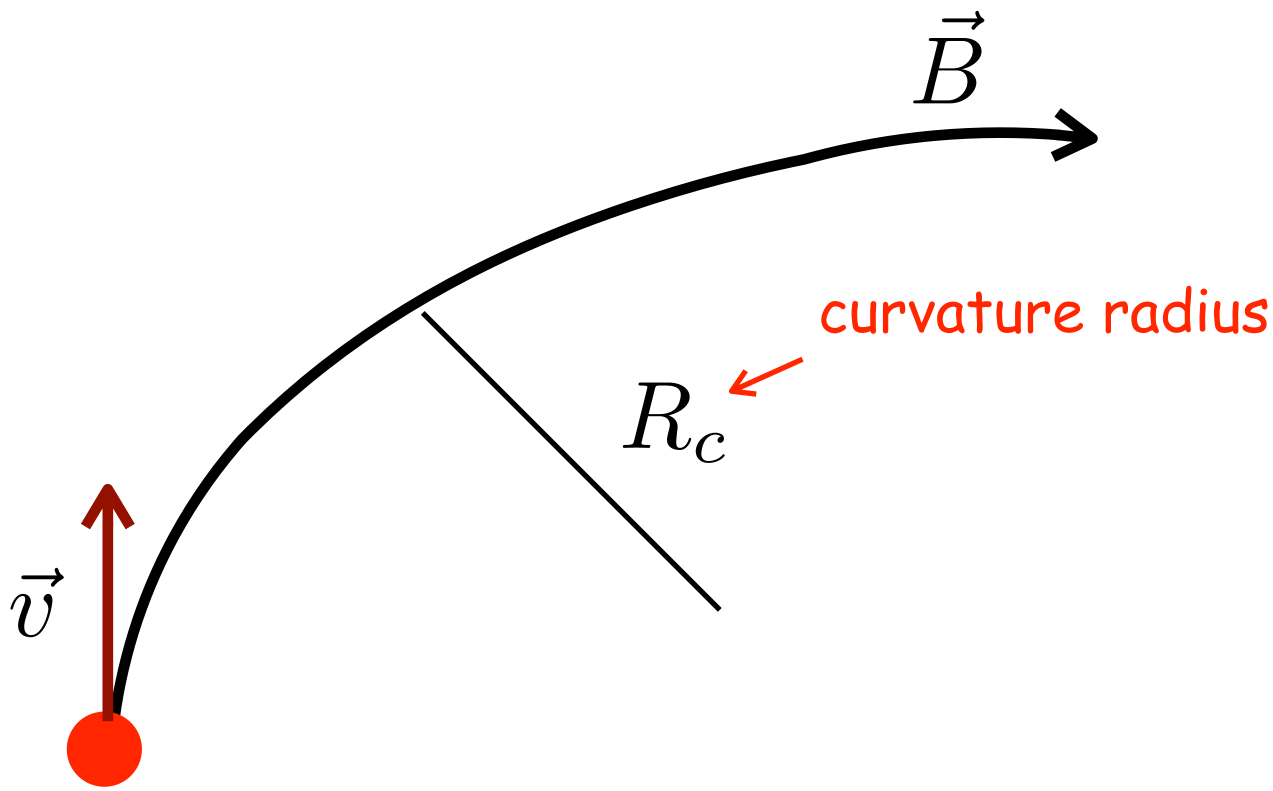
$$P = \frac{2}{3}\left(\frac{\overbrace{q}^{\text{particle charge}}}{\underbrace{m}_{\text{particle mass}}c^2}\right)^4 c\,\overbrace{E^2}^{\text{particle energy}}\,\underbrace{B_{\perp}^2}_{\text{magnetic field orthogonal to particle velocity}}$$

Curvature radiation

radiation emitted by particles that move along curve field lines

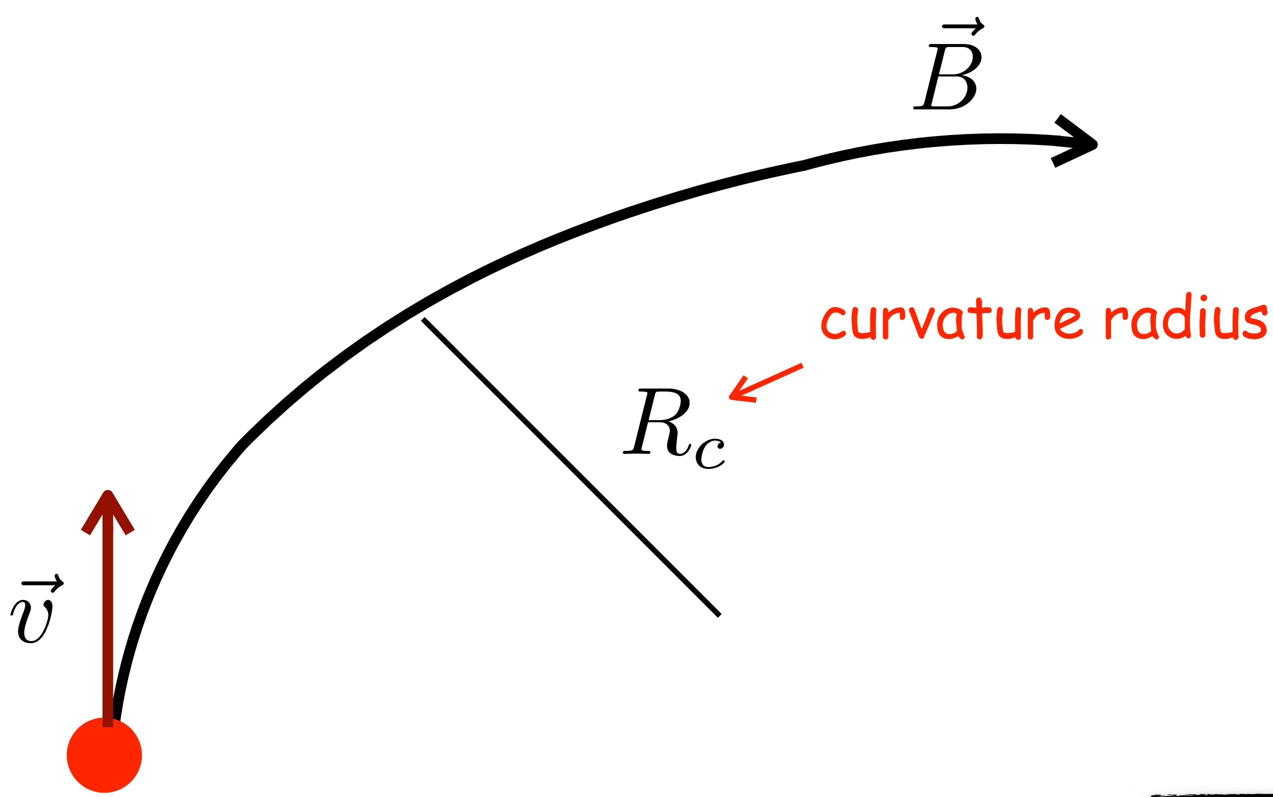
Curvature radiation

radiation emitted by particles that move along curve field lines



Curvature radiation

radiation emitted by particles that move along curve field lines

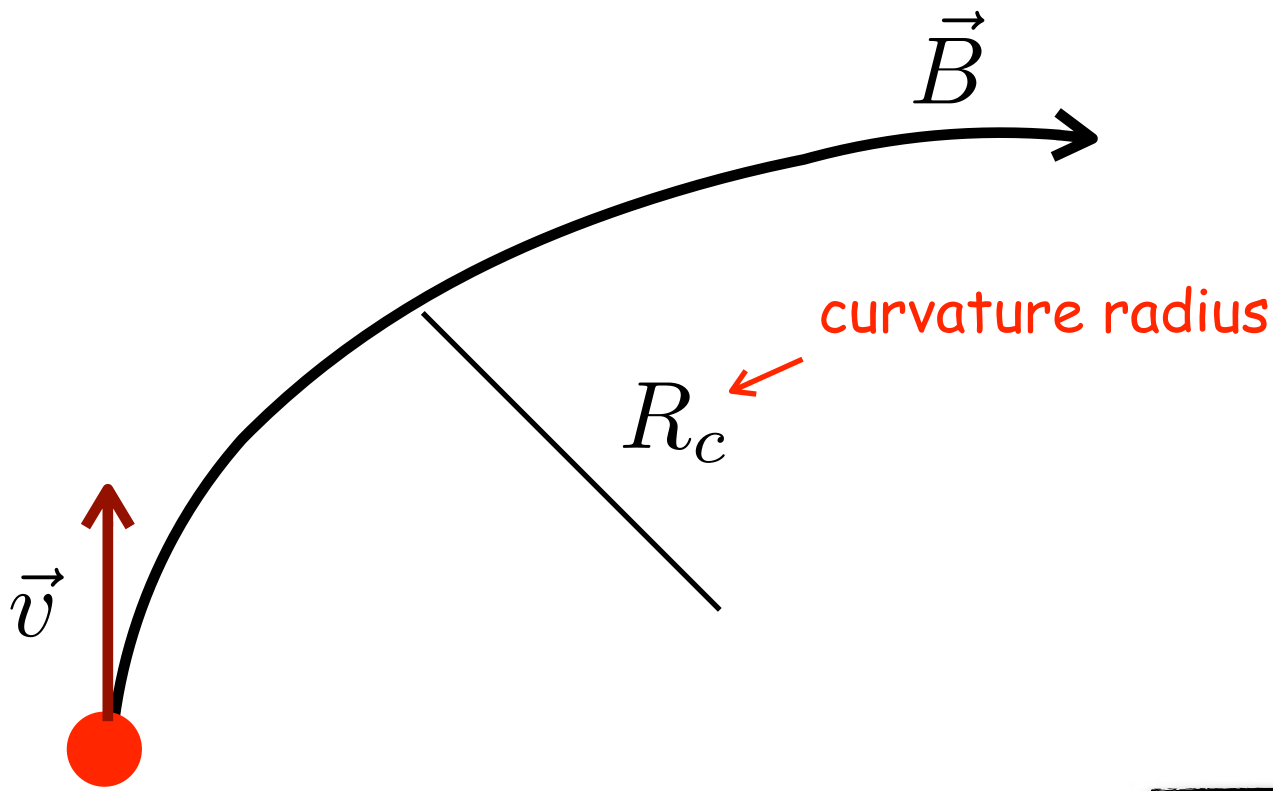


synchrotron

$$P \sim \left(\frac{q}{mc^2}\right)^4 cE^2 B_\perp^2$$

Curvature radiation

radiation emitted by particles that move along curve field lines



synchrotron

$$P \sim \left(\frac{q}{mc^2}\right)^4 cE^2 B_\perp^2 = \left(\frac{q}{m^2c^4}\right)^2 cR_L^{-2}(E)E^4$$

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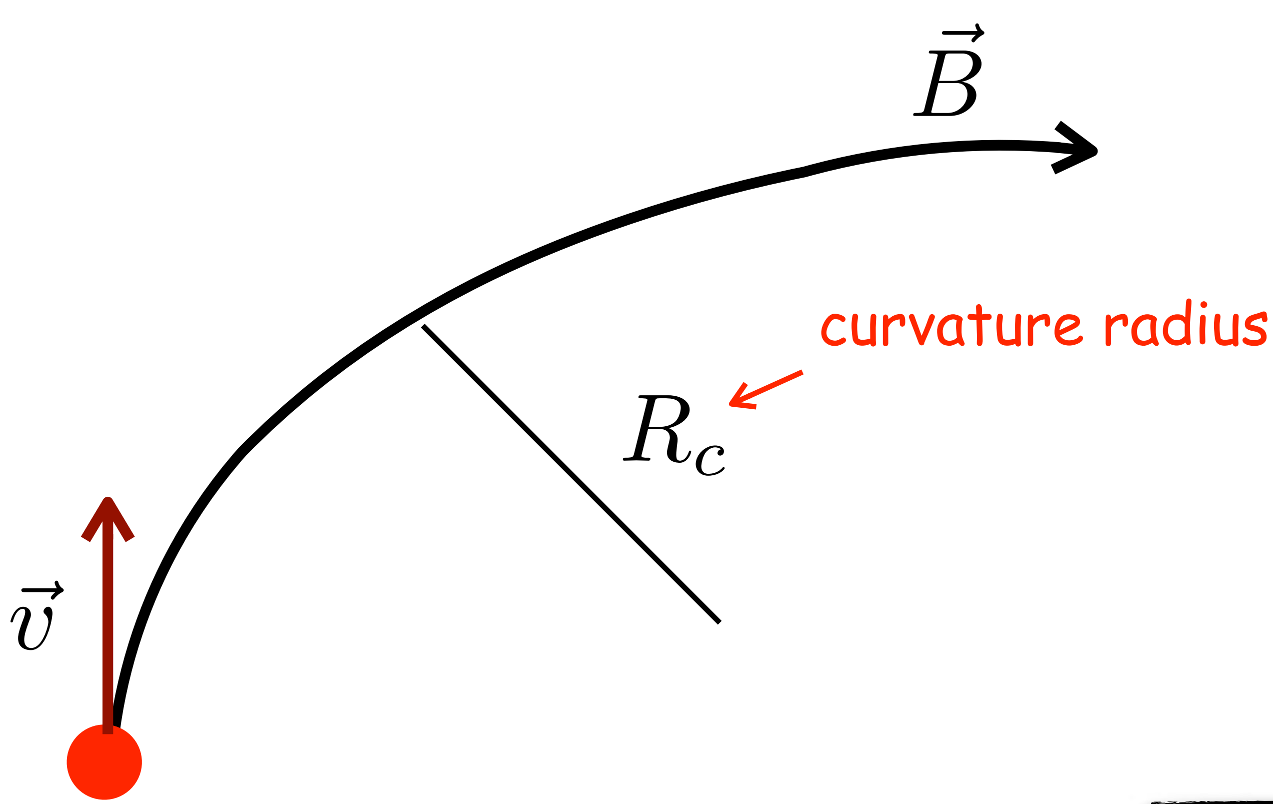
radiation emitted by particles that move along curve field lines



~~synchrotron~~ curvature

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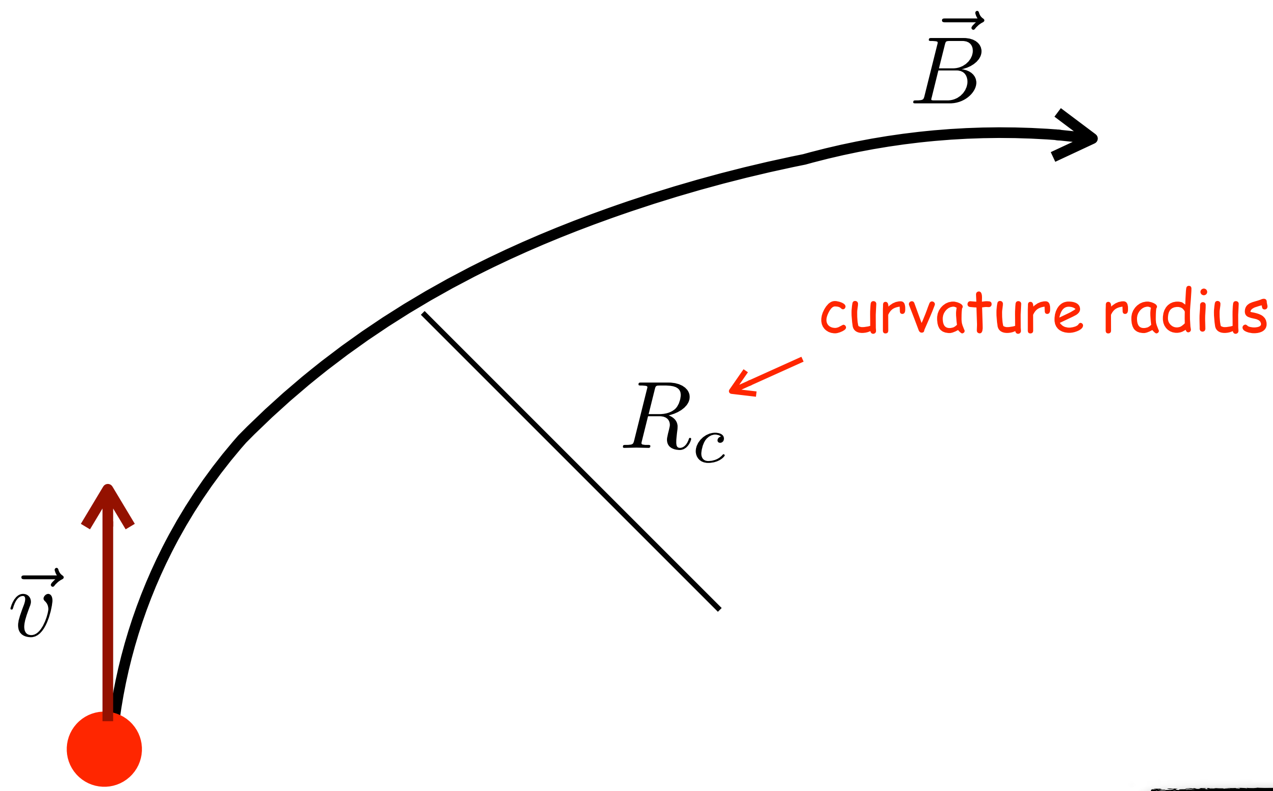

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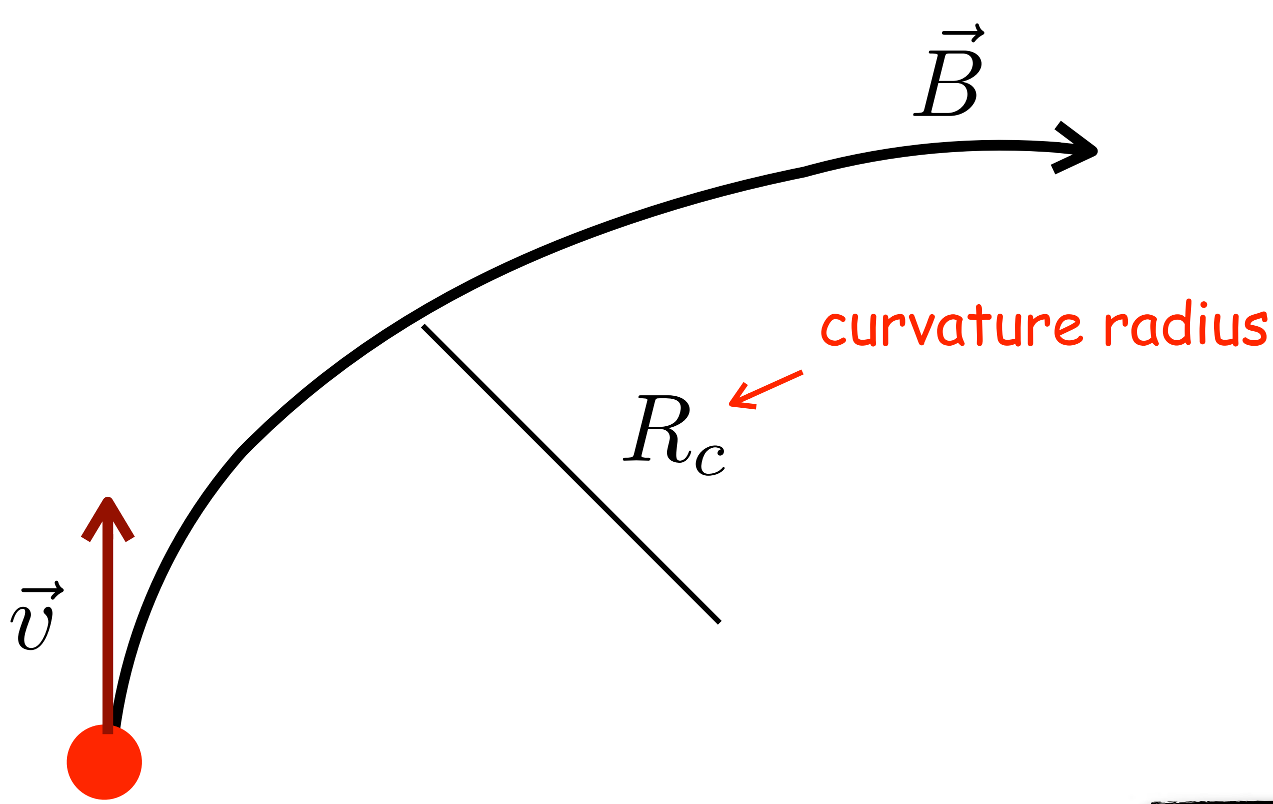
it does not depend on B!

Curvature radiation

radiation emitted by particles that move along curve field lines



relevant for very strong B-fields (e.g. close to pulsars), when any initial component of v orthogonal to B is damped due to synchrotron losses

~~synchrotron~~ curvature

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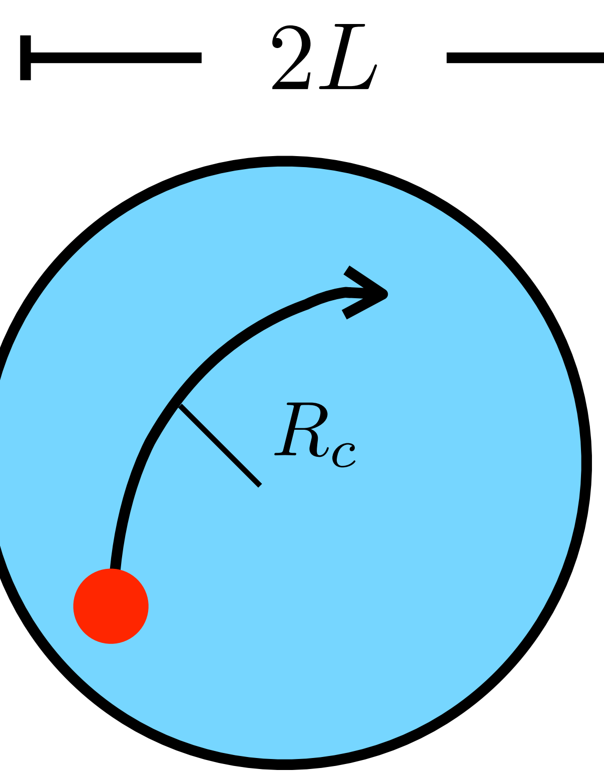
We search for a good accelerator, so in order to minimise radiative losses we set:

$$R_c \approx L$$



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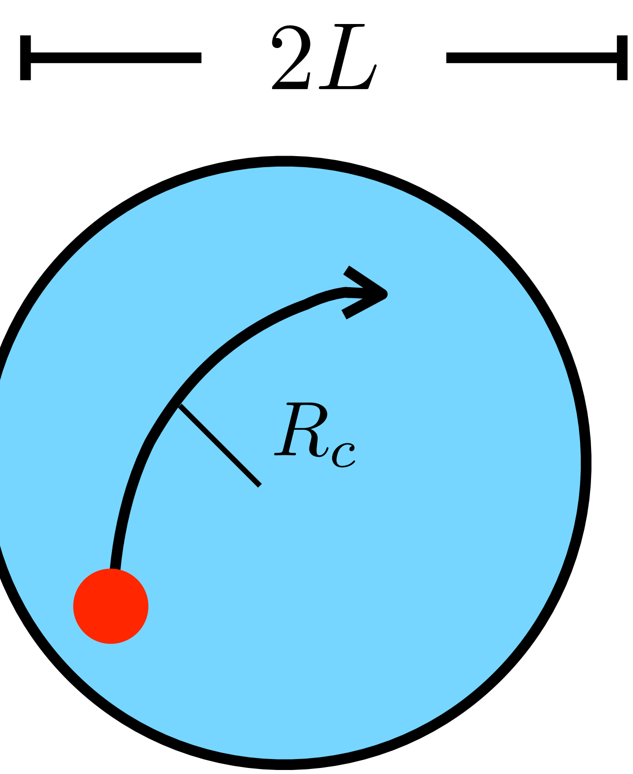

$$\boxed{P_c \sim \frac{q^2 c}{L^2}\left(\frac{E}{mc^2}\right)^4}$$

Minimal (but unescapable) radiative losses

Let's generalise Hillas' considerations by including the effect of radiative losses

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Minimal (but unescapable) radiative losses

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  - matter

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- Magnetic fields are necessary in order to have acceleration

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  - so let's imagine an accelerator where the interactions between accelerated particles and matter and radiation can be neglected

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  - matter
  - radiation
  - magnetic fields
- Magnetic fields are necessary in order to have acceleration
  - so let's imagine an accelerator where the interactions between accelerated particles and matter and radiation can be neglected
- Energetic charged particles in a magnetic field lose energy because of:

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Synchrotron versus curvature losses

$$\frac{P_c}{P_S} = \frac{E^2}{q^2 B_\perp^2 L^2}$$

Aharonian el at. PRD (2002)

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$E < E_*$ —> we consider curvature radiation for these particles, as it maximises the value of E_{max}

Aharonian el at. PRD (2002)

Energetics

losses acceleration

$$P_c = qBc \longrightarrow E_{max} = \left(\frac{BL^2}{q}\right)^{1/4} mc^2$$

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Total magnetic energy in the accelerator

atomic number

$$W_B = \frac{L^3B^2}{6} \sim 2 \times 10^{50} \frac{Z^2}{A^8} \left(\frac{E_{max}}{10^{20}\mathrm{eV}}\right)^8 \left(\frac{L}{\mathrm{pc}}\right)^{-1} \mathrm{erg}$$

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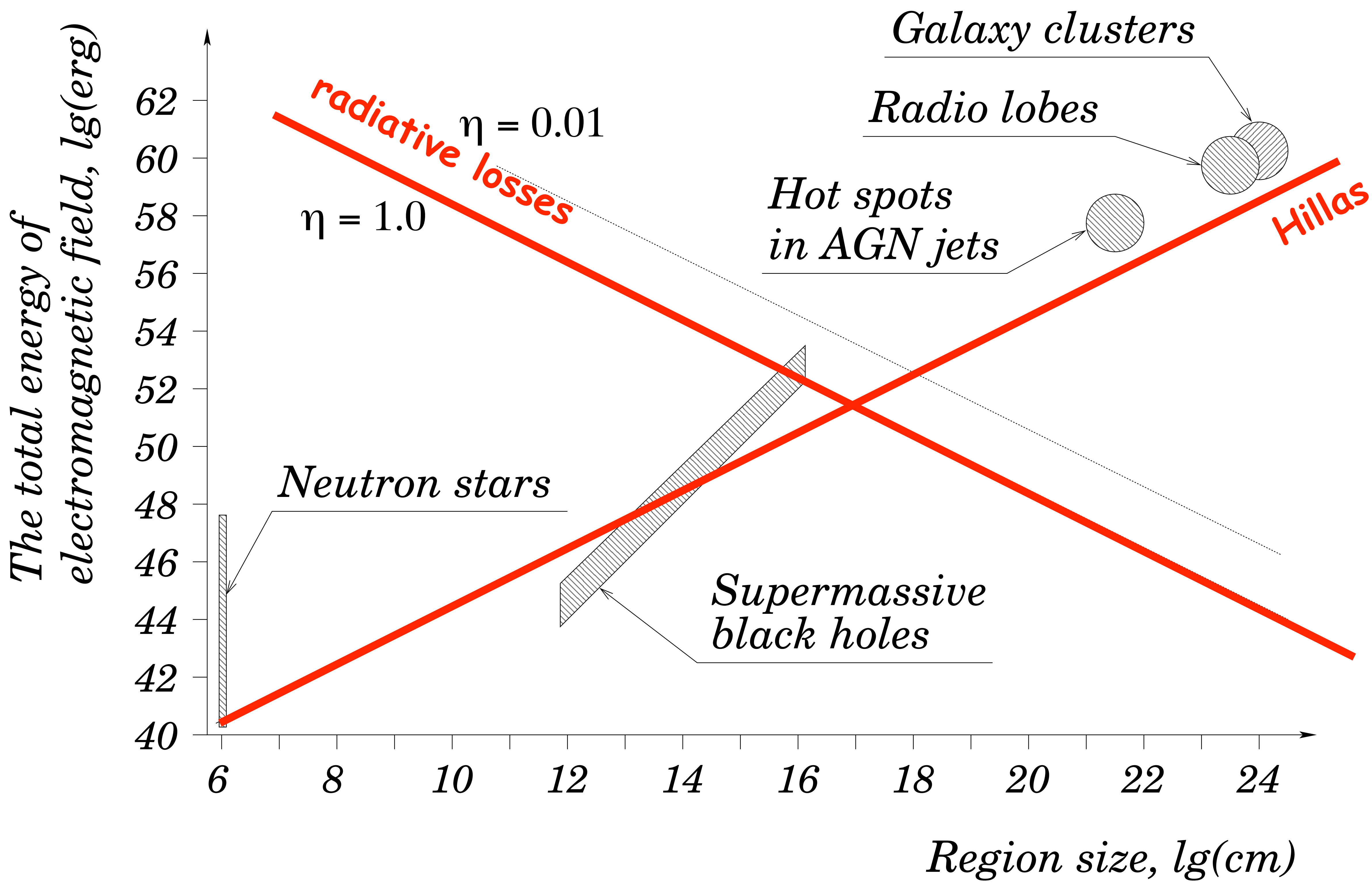
Which accelerators?



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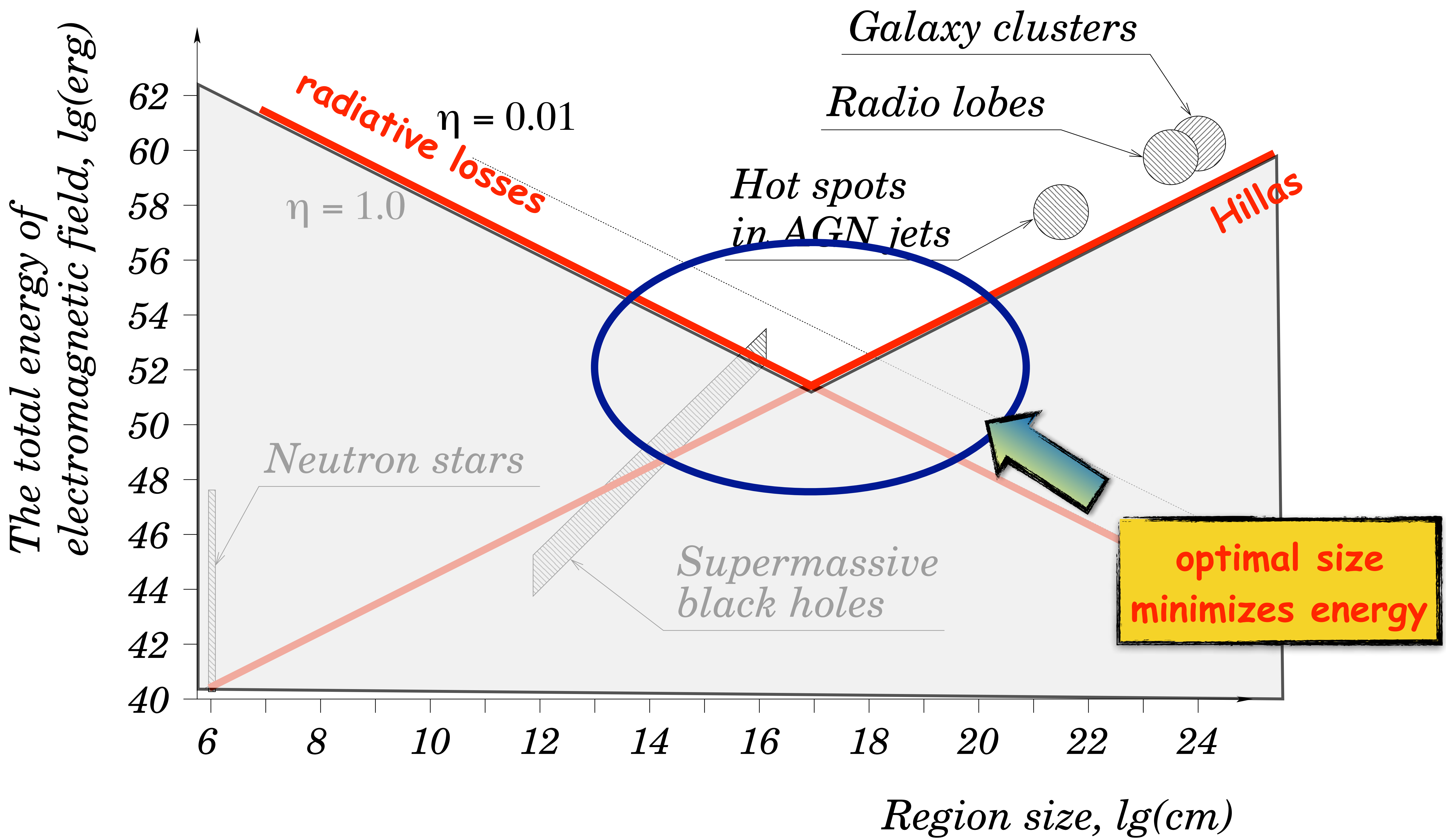

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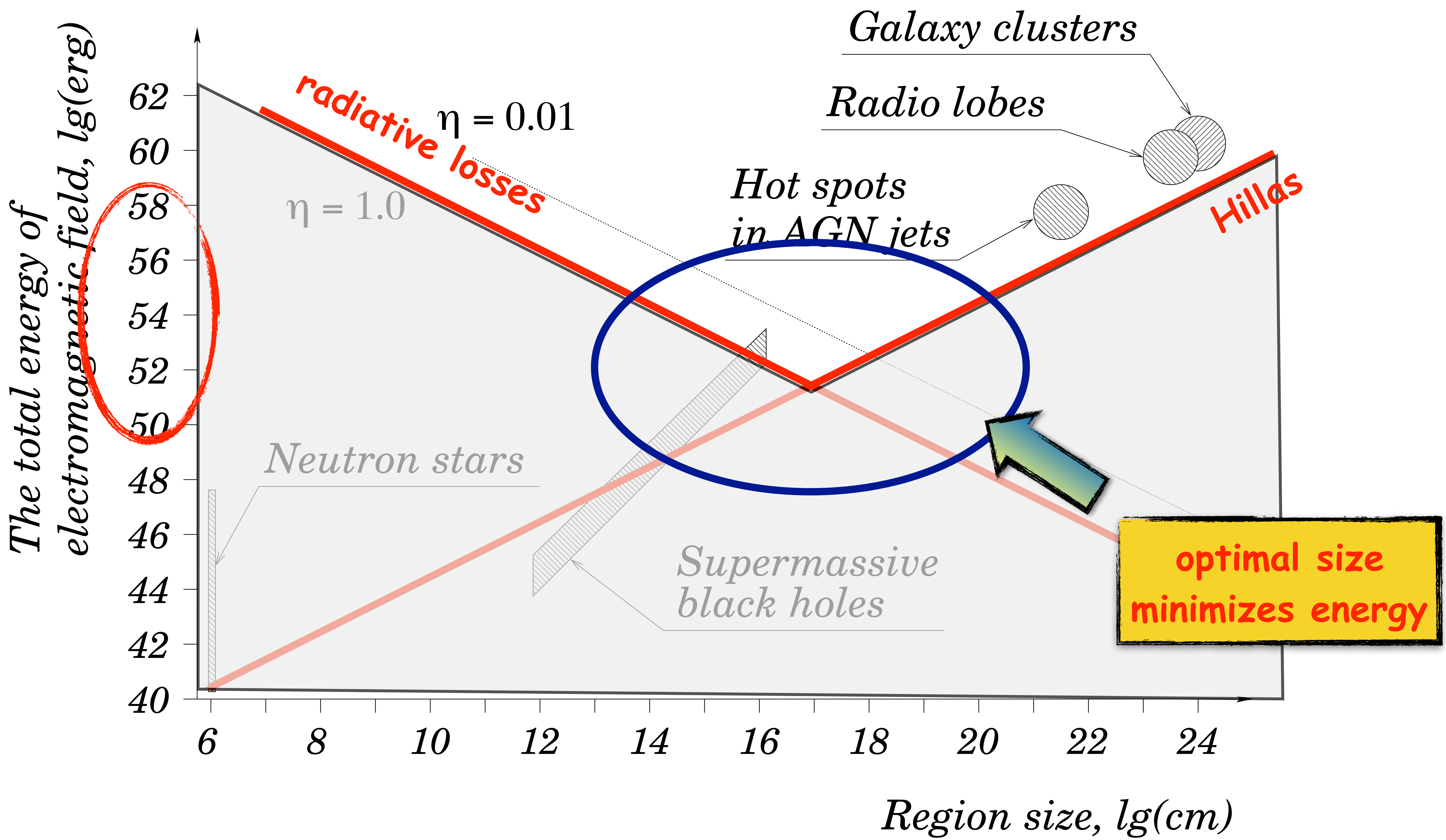

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Optimal size

Hillas | **radiative losses**

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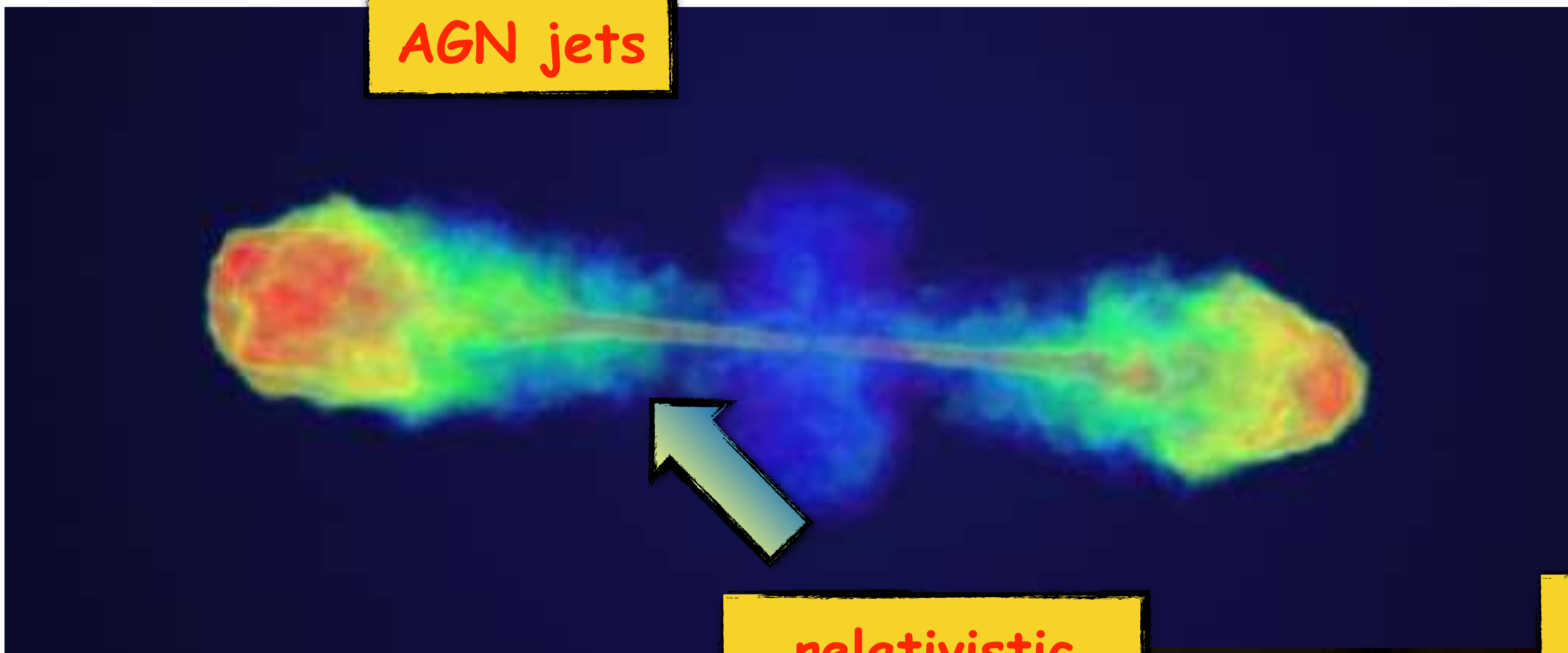
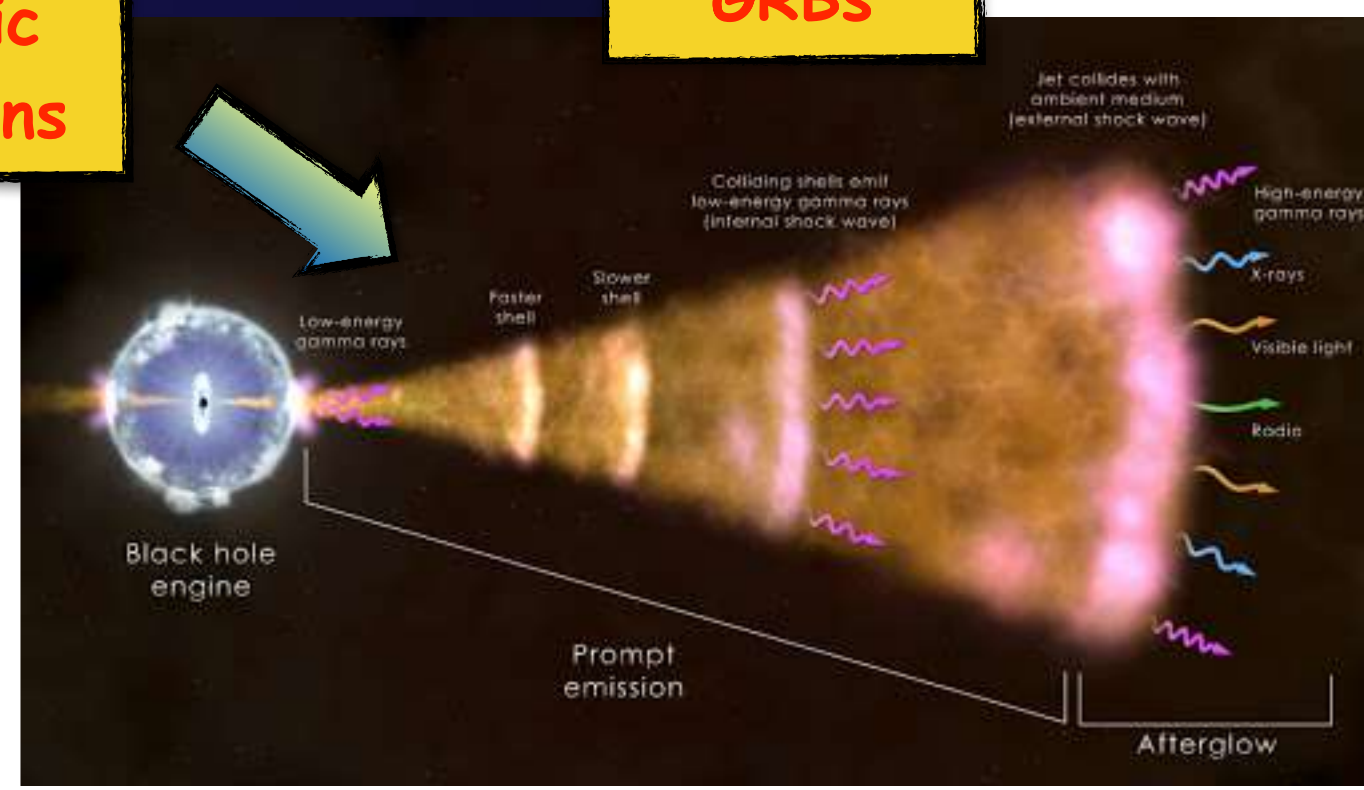
Relativistic sources

AGN jets



relativistic
bulk motions

GRBs

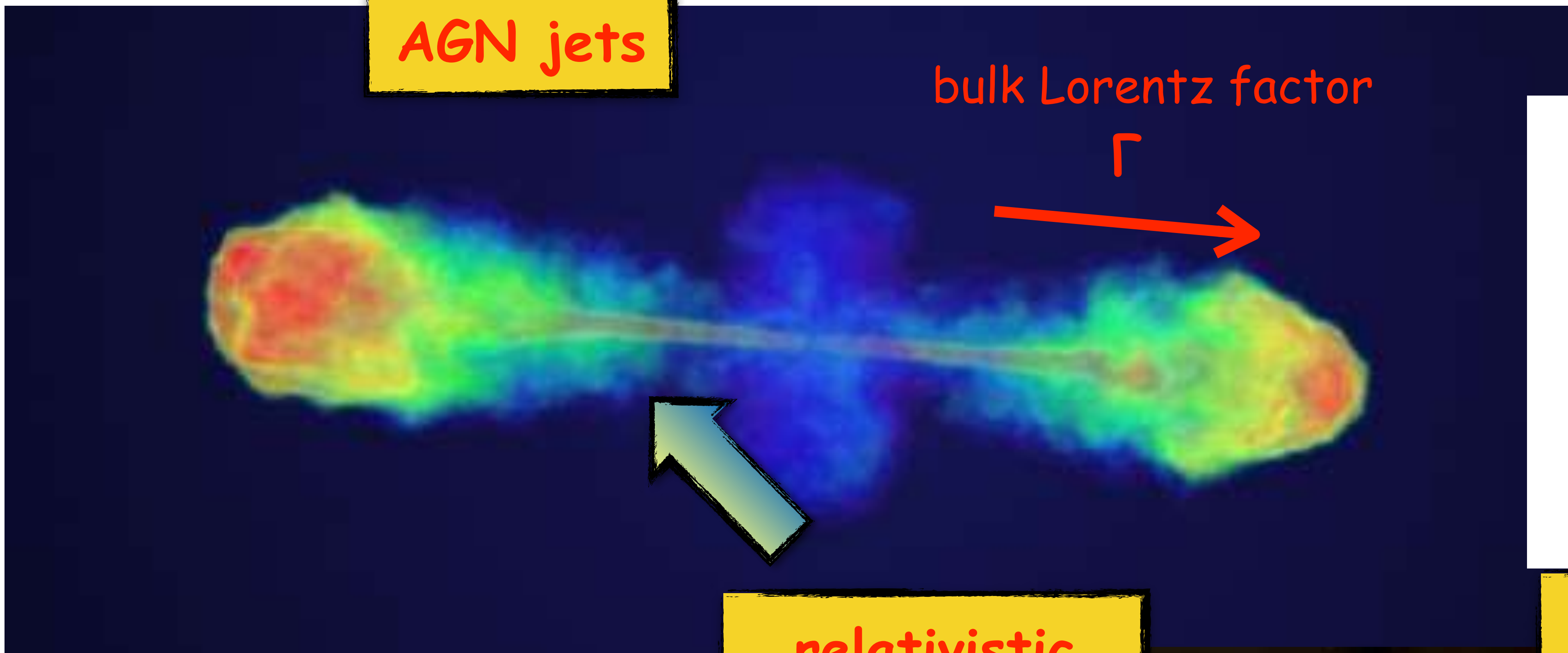


Relativistic sources

AGN jets

bulk Lorentz factor

Γ



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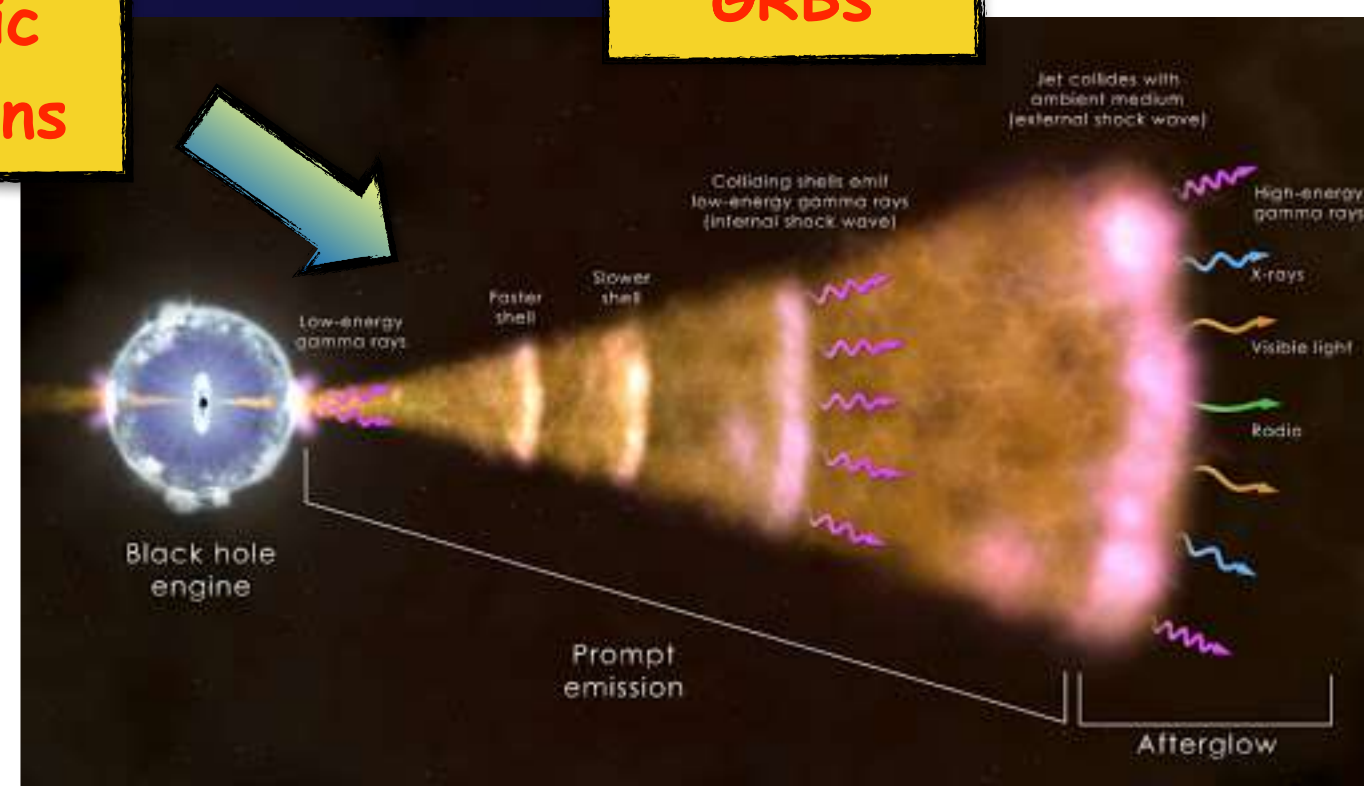

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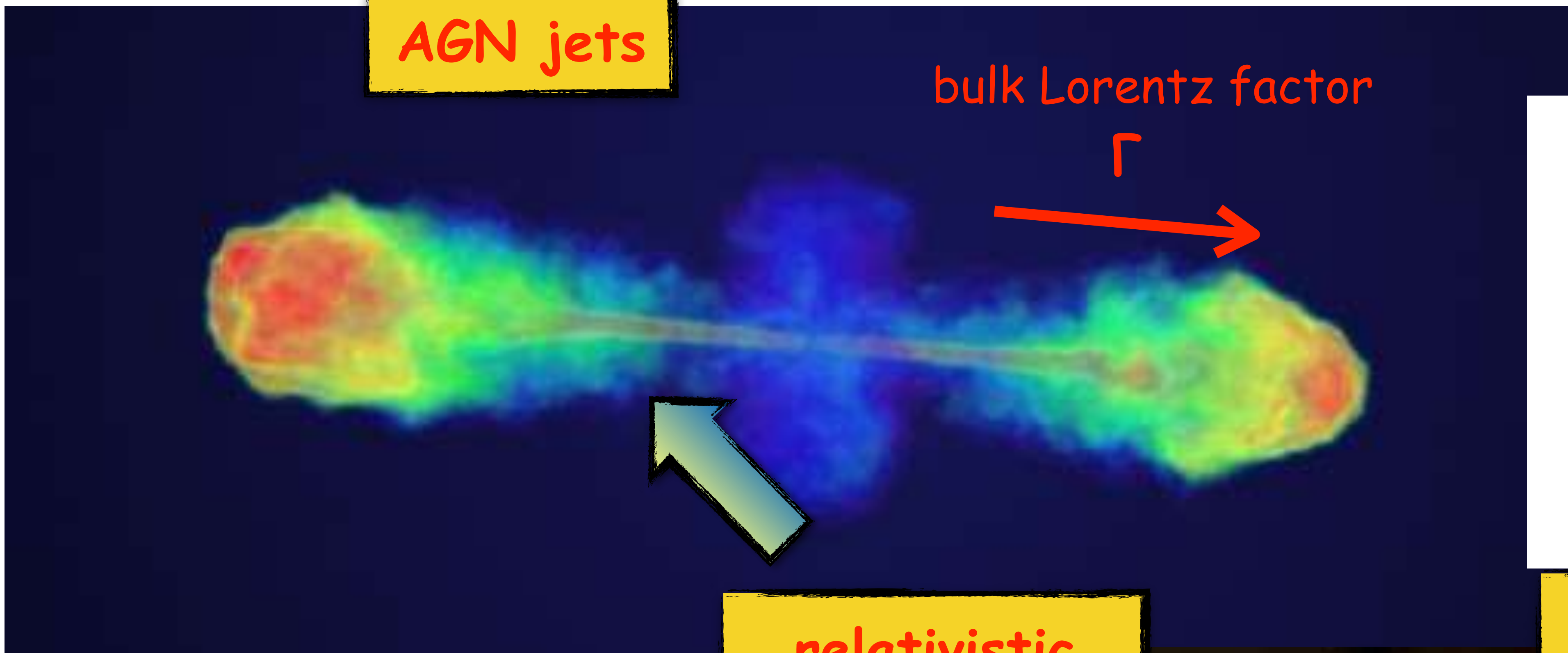

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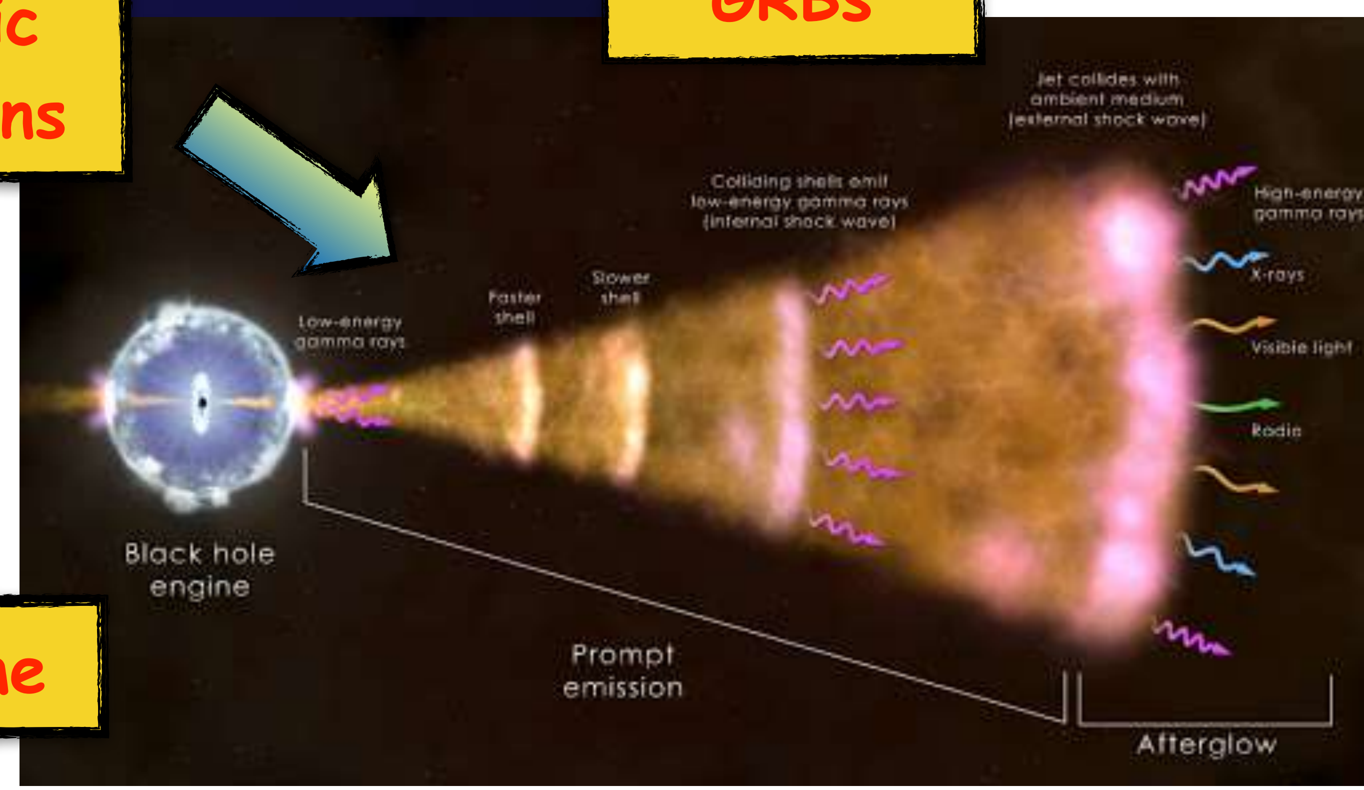
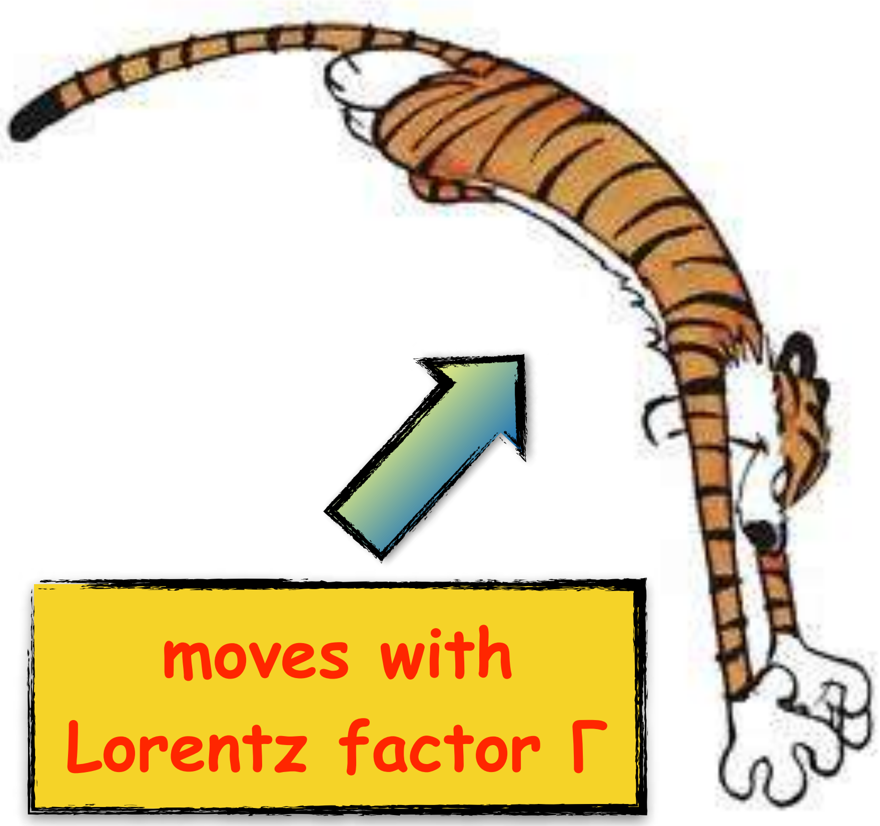


W_B , E_{max} -> lab frame

L' , B' -> comoving frame

Change rest frame



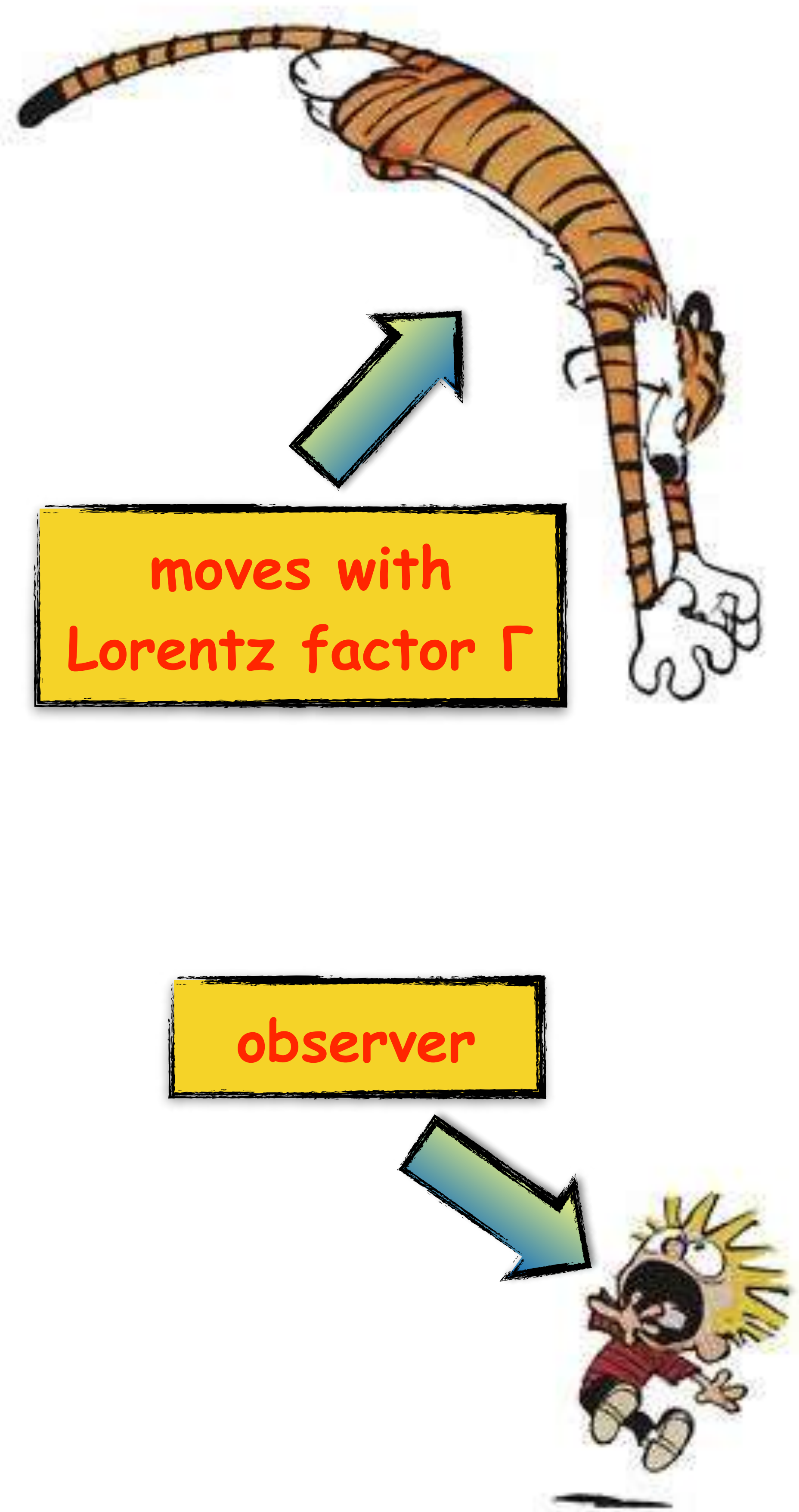
moves with
Lorentz factor Γ

observer

Lee Jones

Change rest frame

$$E_{max} \sim \Gamma E'_{max}$$



Lee Jones

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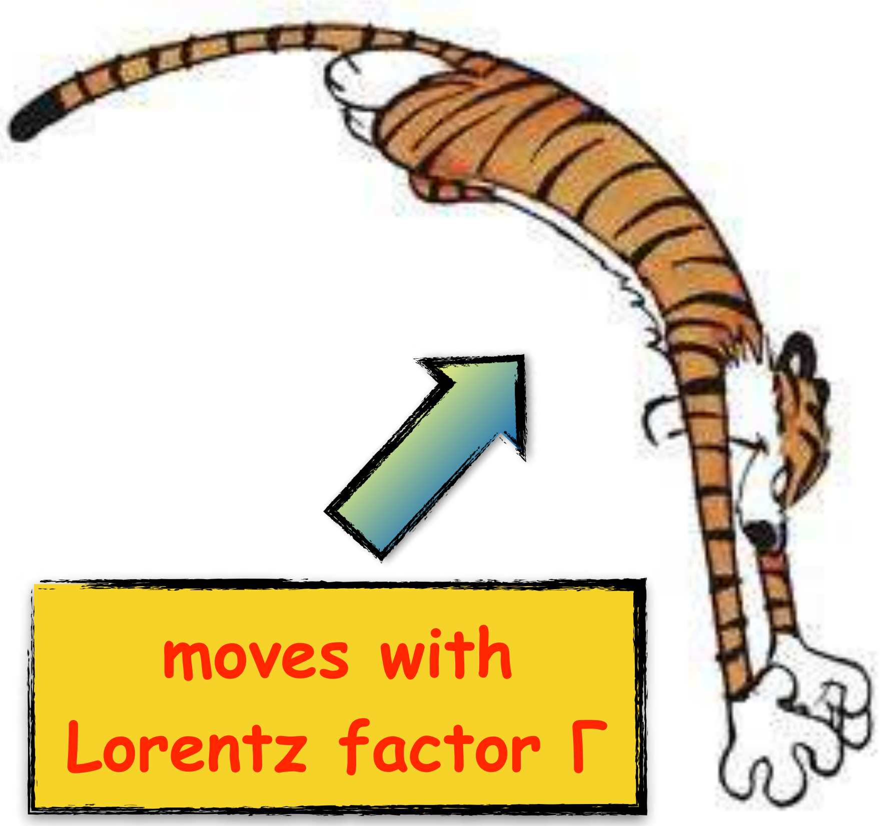

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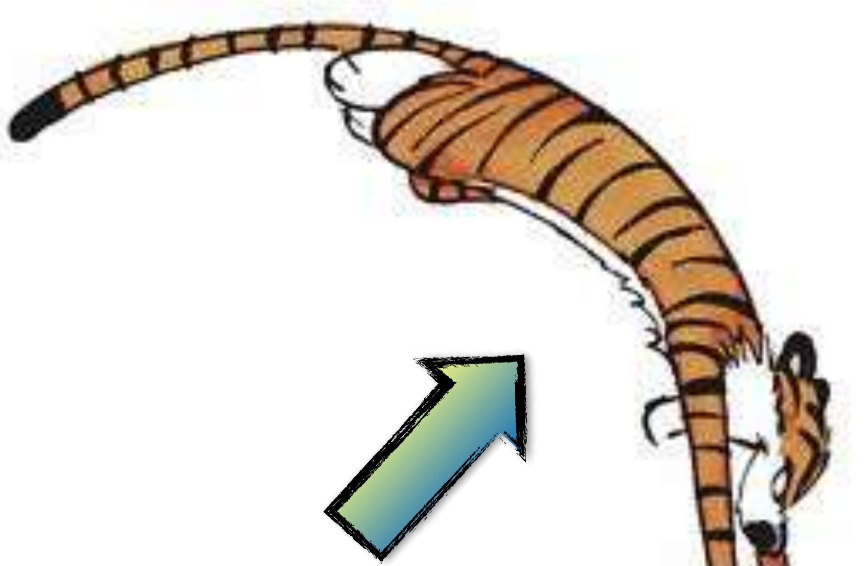

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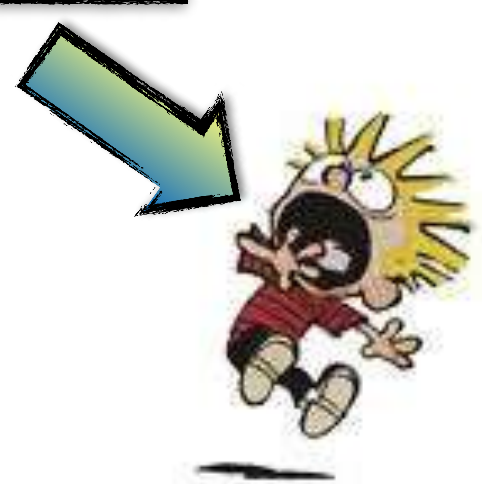

Lee Jones

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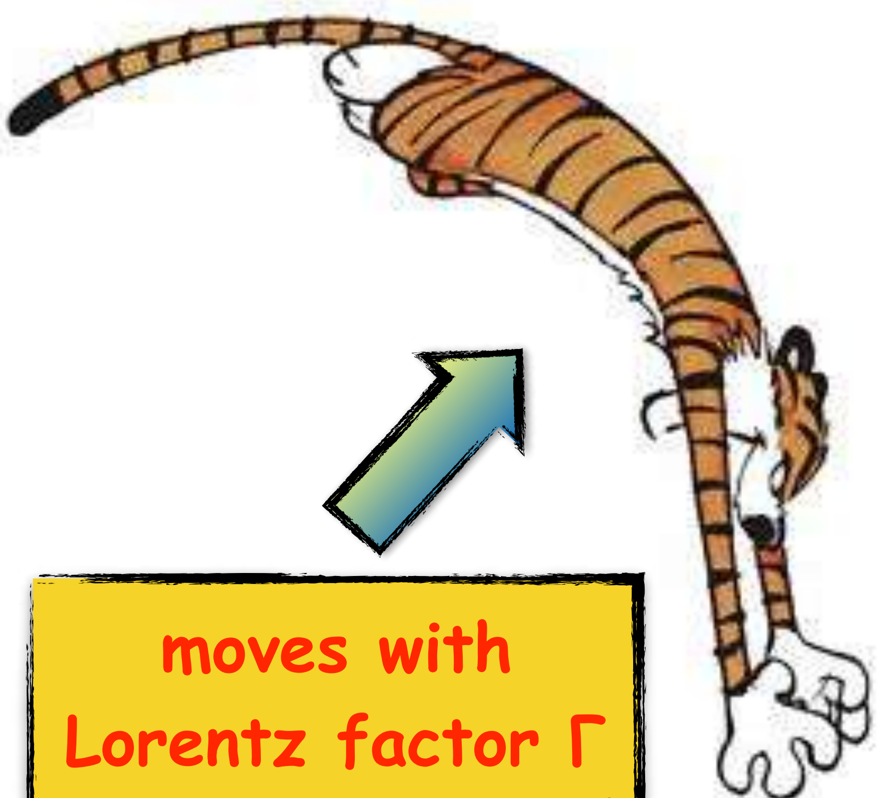

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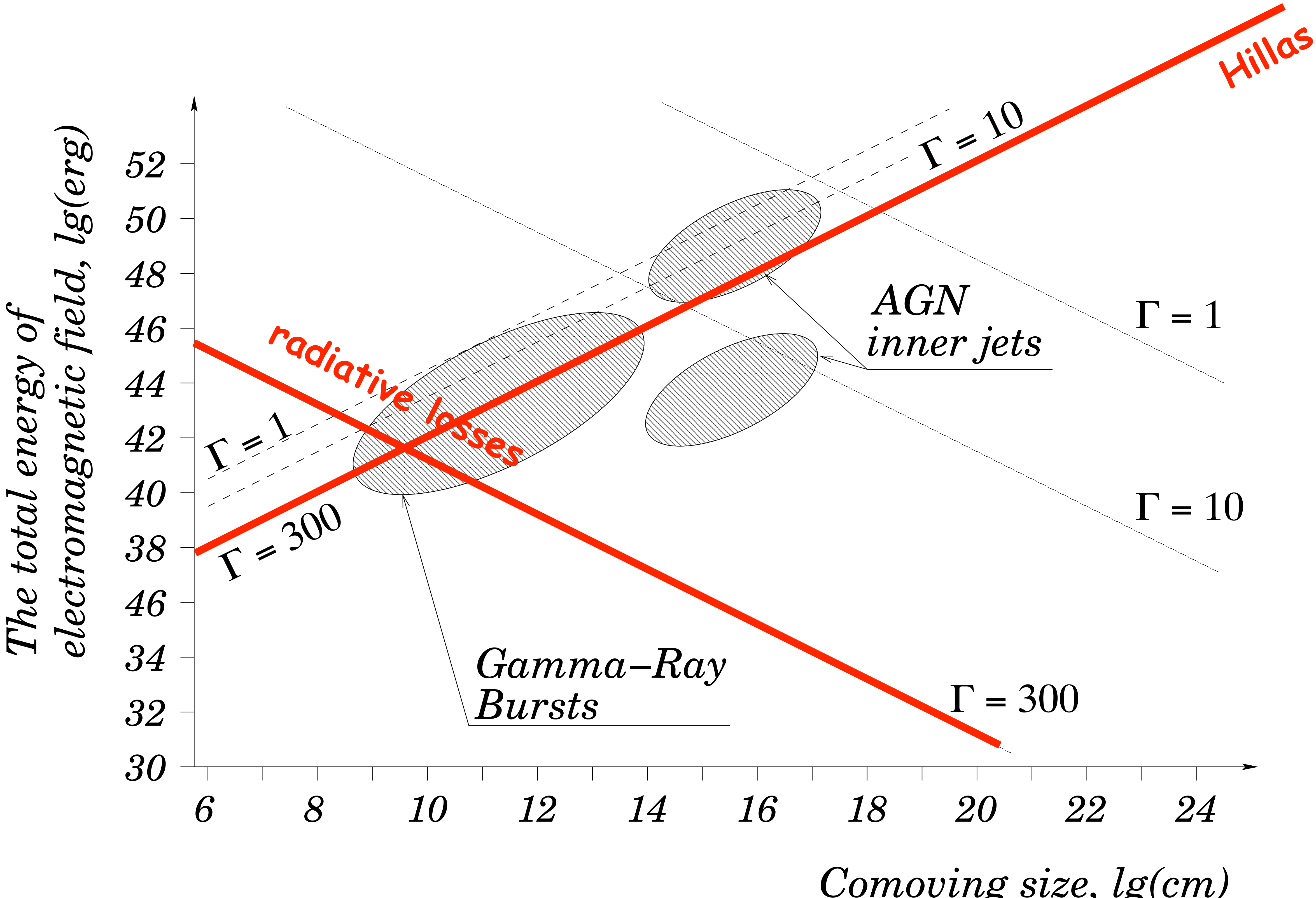

Aharonian el at. PRD (2002)

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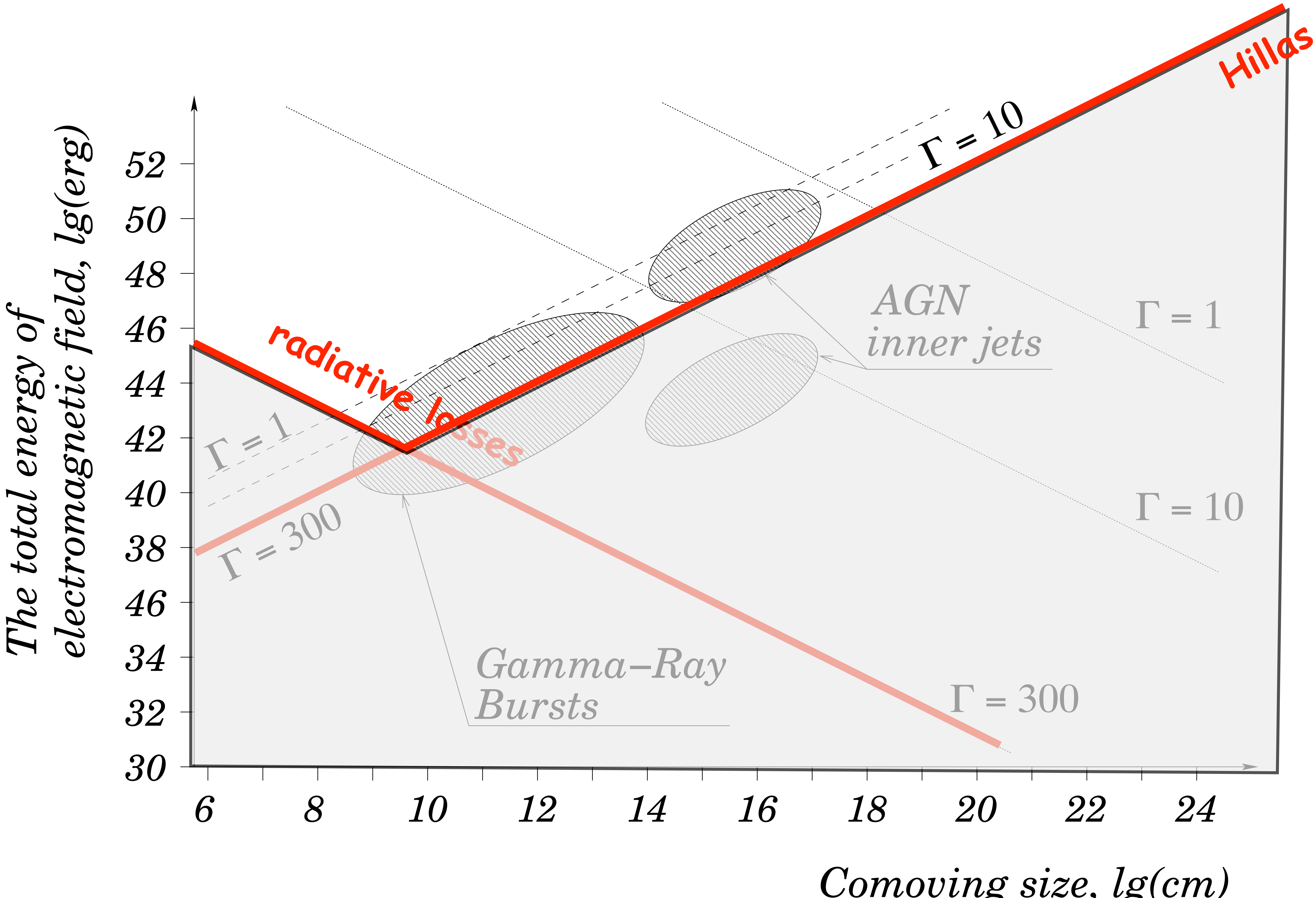

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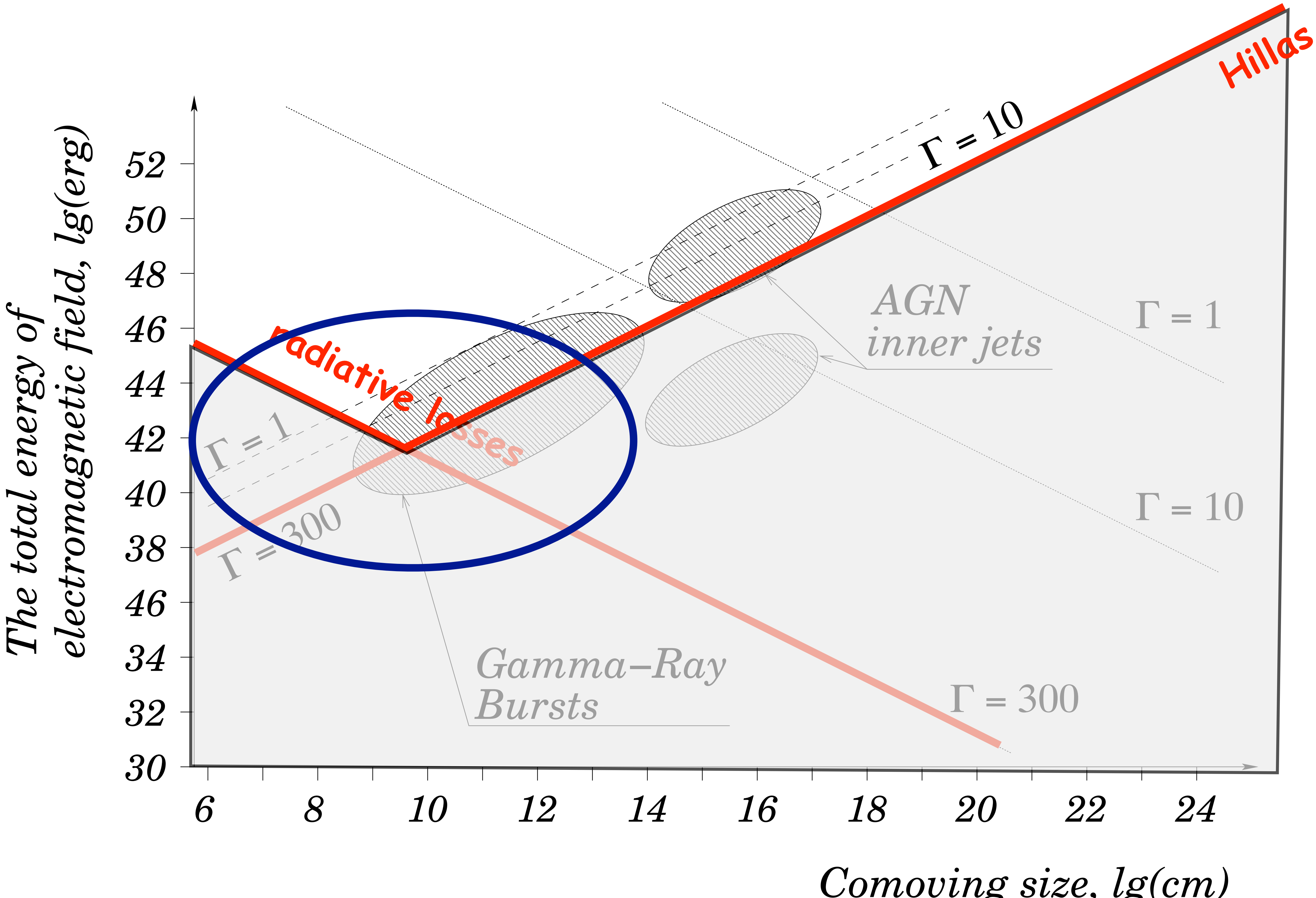

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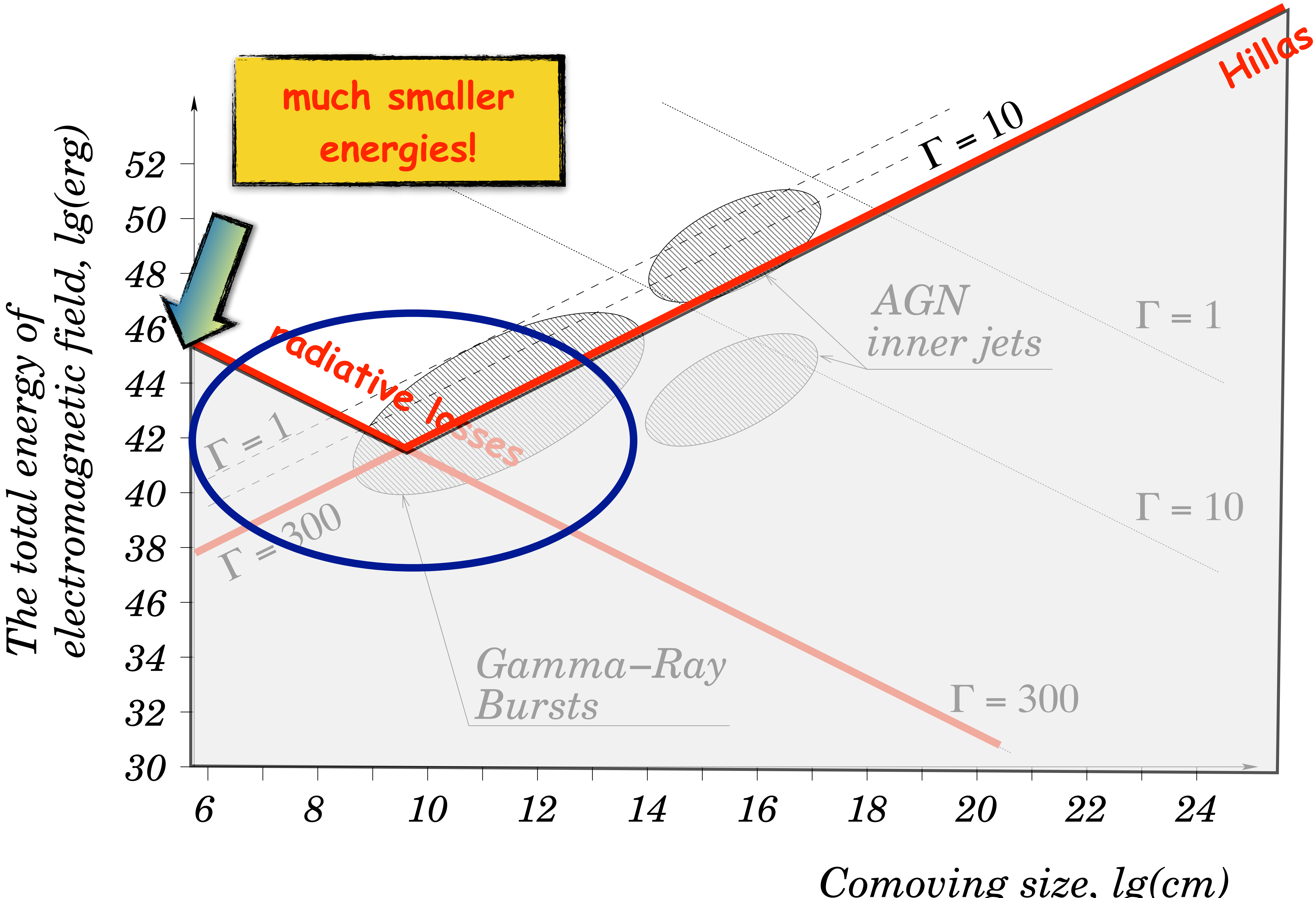

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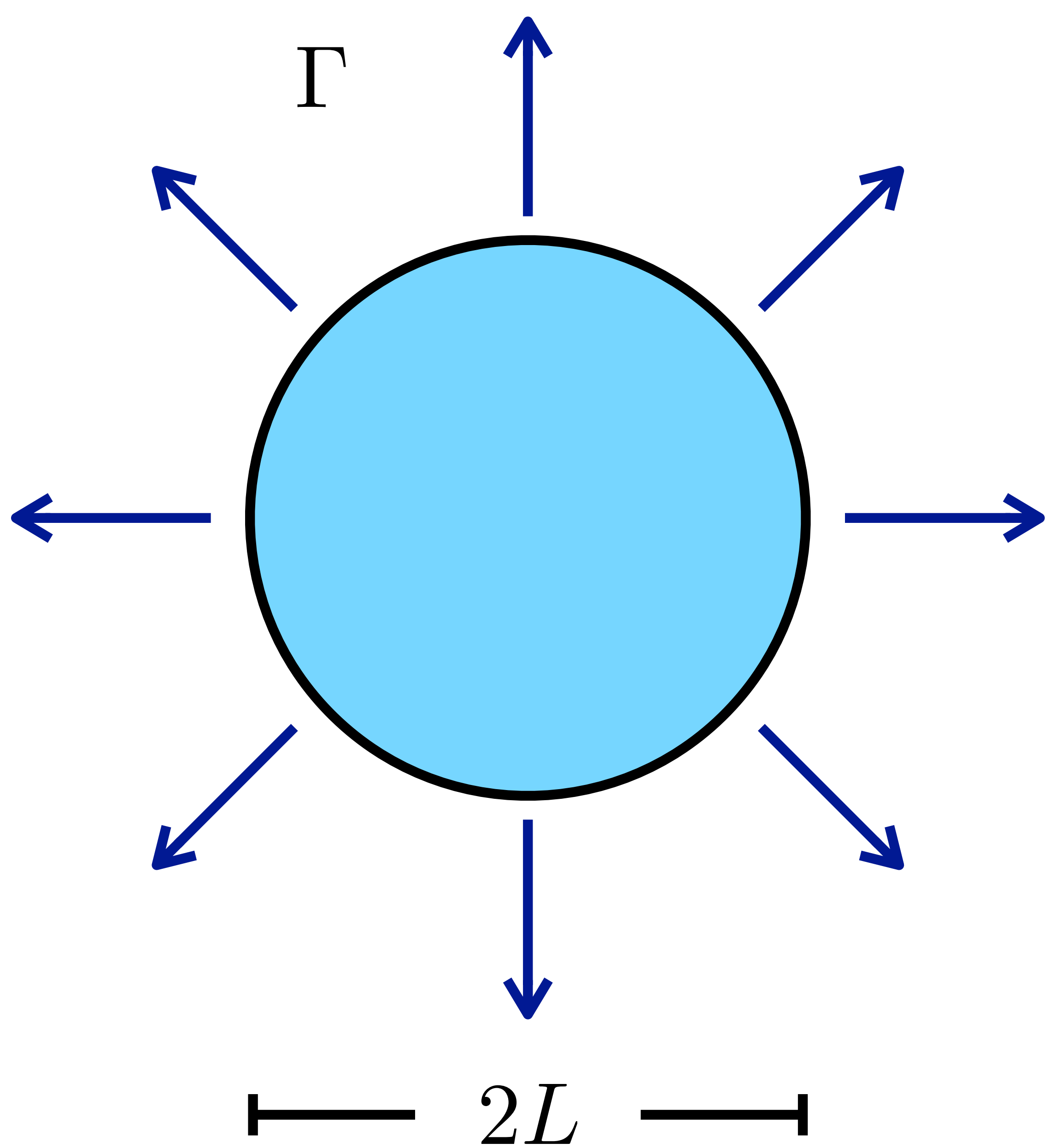
Causality

unfortunately the situation is not as simple as that…
consider for example a wind-type accelerator



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- So we have a limited time to accelerate particles
- Let's say the field drops when:

$$\Delta L \approx L$$
$$t = \Delta L / c \sim L / c$$

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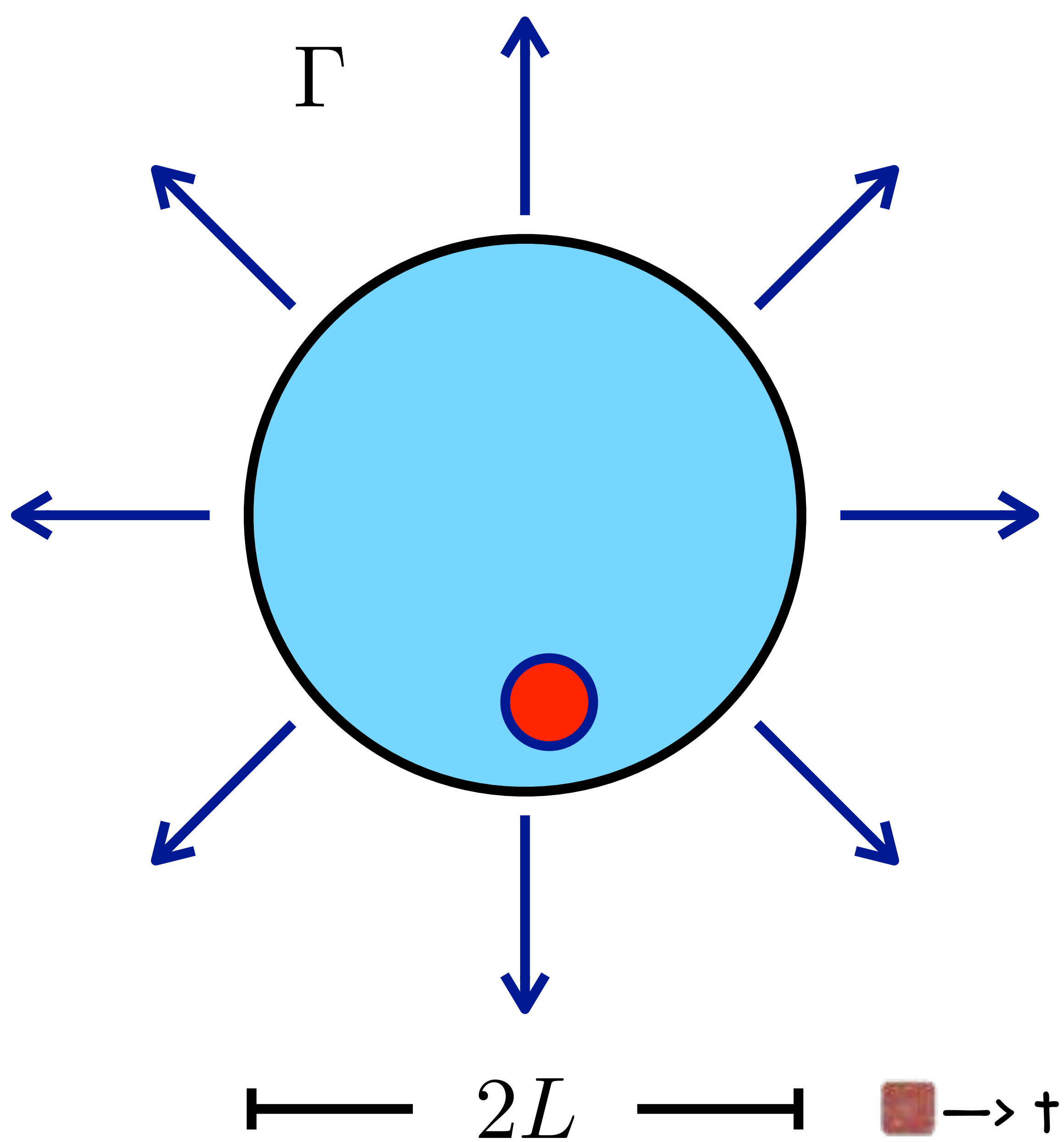

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- A particle moving will the flow can be accelerated over a time (region):

$$t' \sim L/\Gamma c$$
$$R' \sim L/\Gamma$$

- —> the constrain refers to a very small fraction of the volume of the source —> the total energy is much larger!

Things to remember

Hillas criterium

- Combined with constraints due to radiative losses can be used to identify possible particle accelerators
- Even in the most optimistic scenarios, large energy budgets are required to reach 10^{20} eV —> extreme accelerators
- Relativistic effects may, under certain conditions, mitigate the requirements. Otherwise very large sizes have to be invoked.

Summary

Hillas, Hillas, and Hillas
(1) Hillas review paper

1. Particles may gain energy gradually by numerous encounters with regions of changing (moving) magnetic field; such processes are variants of Fermi's mechanism (43). Their advantage is that the energy is spread over many decades, and in the shock-wave variant (7, 8, 12, 64) the spectrum very convincingly emerges as $\sim E^{-2}$. Their disadvantages are that they are slow, and that it is hard to keep up with energy losses at the highest energies.
2. Particles may be accelerated directly to high energy by an extended electric field (e.g. emf arising in rapidly rotating magnetized conductors, such as neutron stars or supermassive objects). Such a mechanism has the advantage of being fast, but it suffers from the circumstance that the acceleration occurs in an environment of very high energy density, where new opportunities for energy loss exist. In addition, the complexity of such an analysis is daunting: and it is usually not obvious how to get a power-law spectrum to emerge.

A. M. Hillas, Annual Review of Astronomy and Astrophysics, 22 (1984) 425

Hillas, Hillas, and Hillas
(2) Hillas criterium

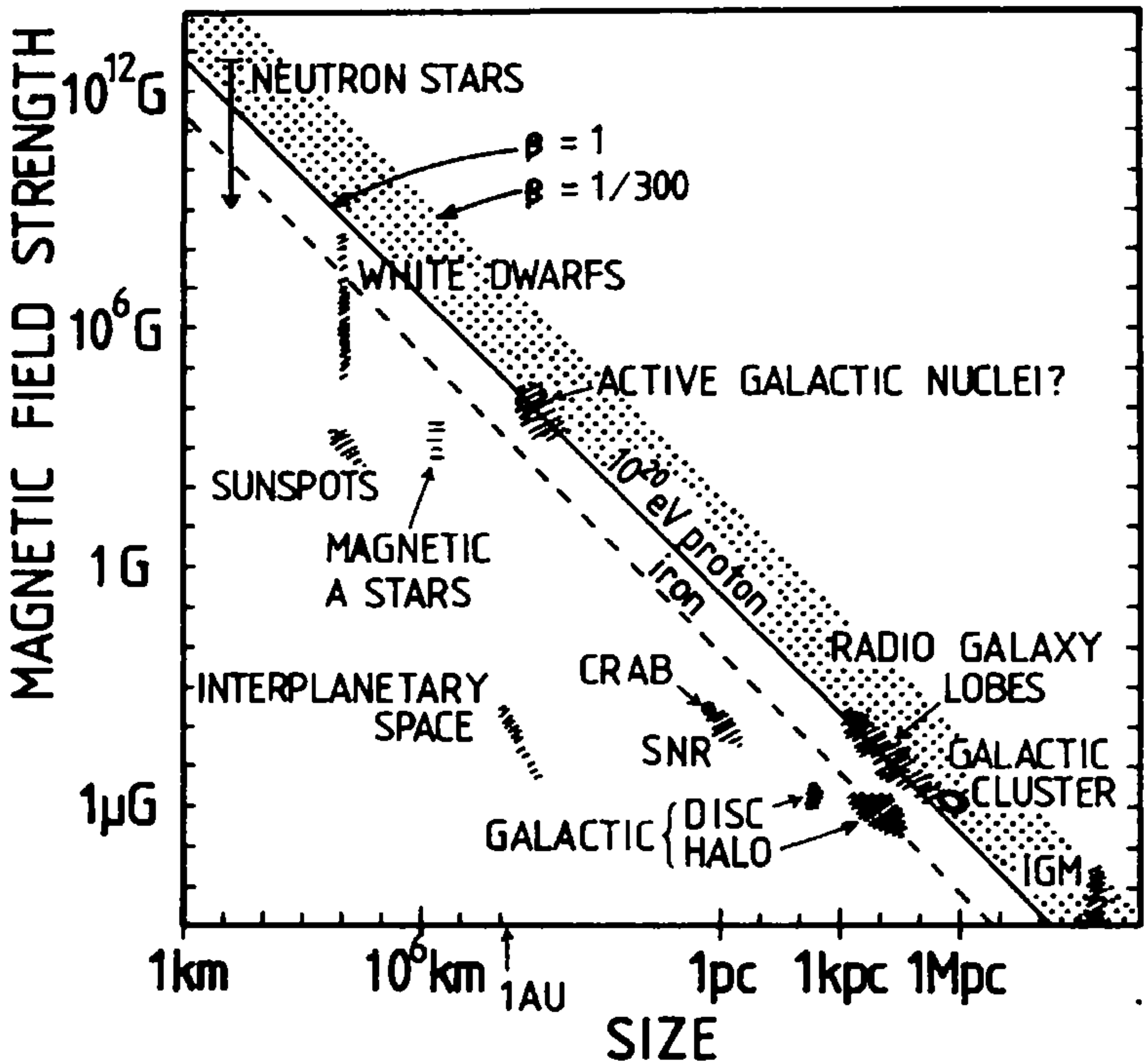
Hillas criterium

$$E_{max} \lesssim qB\frac{u}{c}L \quad \longrightarrow \quad E_{max} \lesssim qBL \quad \longrightarrow \quad R_L(E_{max}) \lesssim L$$

relativistic motion

$$E_{max} \lesssim \Gamma qBL$$

Hillas, Hillas, and Hillas
(3) Hillas plot



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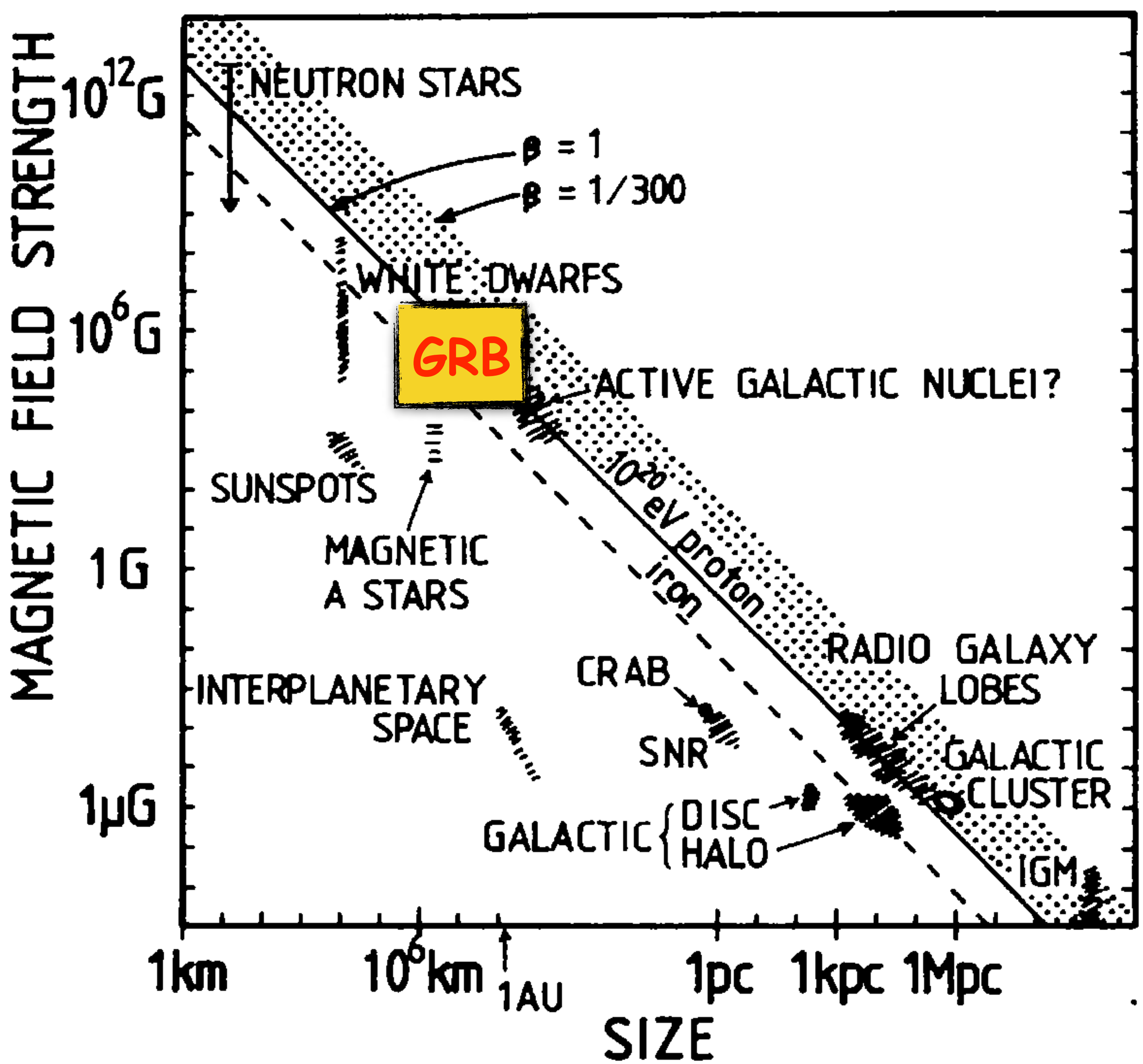
Hillas, Hillas, and Hillas
(3) Hillas plot (updated)



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The End

Thanks to everybody